L. M. Berkeley

Great Circle Sailing.

GREAT CIRCLE SAILING

BY

L. M. BERKELEY

Author of "North Star Navigation"

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Author of "North Star Navigation"

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PREFACE.

In a number of recent works on navigation there is a tendency to underestimate the importance of great circle sailing. The authors of these books consider the calculation of a great circle course to be so complex a process that the resulting advantages do not compensate for the time expended; and these advantages are declared to be but slight after all.

The object of this little work is to controvert the above views in both the aspects mentioned, and to show (1) that the calculation of a great circle course may, for practical purposes, be made so simple as to take only a few minutes of time, and (2) that in these days of monster steamships, where every hour saved in the length of a voyage means a substantial economy, great circle sailing has attained an importance which no navigator can afford to ignore.

Works on navigation usually give full consideration to the problem of calculating the initial great circle course from the starting point to the port of destination (which is, in fact, only an every-day problem in spherical trigonometry); but, as a rule, they give little or no attention to the further problem of changing the initial course from time to time so as to keep the vessel on the great circle path. In the following pages, an attempt is made to supply this deficiency; and a formula is given (which, so far as the writer is aware, is entirely new) for altering or correcting the initial course as frequently as the present facilities for steering vessels will allow.

It is probable that practical seamen will criticise the following pages as too mathematical. If so, the author will admit the charge at once. However, he would fain plead, by way of confession and avoidance, that this railing accusation should be brought against the subject, and not against him. He

would further plead, by way of rebutter, that the practical seaman does not really need to pay any attention to the mathematics. Let him jettison the whole of it, and turn at once to the worked examples on pages 13 and 26. These examples cover every case that can arise in practice, and the practical man can learn in a very few minutes all that is really needed to be known. If by this means he can bring his vessel to port five or six hours ahead of time on each trans-Atlantic voyage, his time will not have been wasted; for he will thereby save his owners a sum which, in the case of the largest vessels, may amount to as much as $100,000 a year (see page 45). Even the adamantine hearts of owners ought to be softened by such a showing.

Acknowledgments are hereby made to Miss Flora H. Mac Ivor of Teachers College for assistance in the preparation of the tables.

L. M. B.

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ABBREVIATIONS AND SYMBOLS.

C = course. C is measured from the north, clockwise (the clock lying on its back, face upward), from 0° to 360°.

dC = increase (or decrease) in course, during any small interval of time, the interval being usually taken so as to make $dC = 1°$. In north latitudes, sailing east, dC is plus; sailing west, dC is minus. In south latitudes, these signs are reversed.

C_m = mean or average value of C during the interval of time in which C increases (or decreases) 1°.

φ = latitude of ship; φ is plus in north latitudes, and minus in south latitudes.

φ' = latitude of ship after interval of time in which C changes 1°; $\cos \varphi' = M \cos \varphi$.

$d\varphi$ = increase (or decrease) in φ, while C is changing 1°; $d\varphi$ may be either plus or minus.

φ_m = mean or average or middle latitude during interval of time in which C changes 1°; $\varphi_m = \dfrac{\varphi + \varphi'}{2} = \varphi + \dfrac{d\varphi}{2}$.

φ_a = latitude of port of arrival.

ds = distance sailed by ship during interval of time in which C changes 1°; $ds = N \cot \varphi_m$.

δ = polar distance = $90° - \varphi$.

P = difference between longitude of ship and longitude of port of arrival. P is plus when measured easterly from ship, and minus when measured westerly.

$M = \dfrac{\sin C}{\sin (C + dC)}$; tabulated in Table 5.

$N = \dfrac{dC}{\sin \left(C + \dfrac{dC}{2} \right)}$; tabulated in Table 5.

CHAPTER I.

ADDITION-SUBTRACTION (GAUSSIAN) LOGARITHMS.

§ 1. Explanation.

As this little work will make much use of addition and subtraction logarithms, it will not be inappropriate to give a brief account of them for the benefit of readers to whom they may be a novelty.

Addition-subtraction logarithms were devised by the great mathematician Gauss, and are often (and properly) called Gaussian logarithms. They furnish the best means of dealing with formulæ which involve the sum or difference of quantities which are themselves unknown, but whose logarithms are given. For example, take the fundamental formula of spherical trigonometry:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

If b, c, and A are known, and it is required to find a, the computer proceeds first to find the logarithm of the product ($\cos b \cos c$), and then the logarithm of the product ($\sin b \sin c \cos A$). From these two logarithms as the data, he can, by means of the table of addition logarithms, find the logarithm of the sum of the two terms ($\cos b \cos c$) and ($\sin b \sin c \cos A$), without reverting in the process to the natural numbers represented by the given logarithms. The immense advantages of such a method are obvious.

So also, if a, b, and c are given, and A is desired, the above equation becomes

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$
$$= \cos a \operatorname{cosec} b \operatorname{cosec} c - \cot b \cot c.$$

Here the table of subtraction logarithms gives at once the logarithm of the difference of the two terms on the right-hand side of the equation, that is, it gives an expression for the value of log cos A on the left-hand side of the equation; and A may then be read off from the table of logarithmic cosines in the usual way.

Tables of addition and subtraction logarithms are given in the appendix. The addition table (Table 3) is constructed as follows:

Each number in column D is the logarithm of some number, say n. The corresponding number in column A is the logarithm of $(1 + 1/n)$. That is, $D = \log\ n$, and $A = \log (1 + 1/n)$.

By means of this table, if we have log a and log b, where a and b are any given numbers, we can find log $(a + b)$ by taking the difference log a − log b, calling this difference D, looking out in column A the number corresponding to D, and adding this number to log a. In other words, if log a − log $b = D$, then log $a + A = \log (a + b)$.

The proof is as follows: If log a − log $b = D$, and $D = \log n$, then log a − log $b = \log a/b = \log n$, and $a/b = n$. But $A = \log (1 + 1/n) = \log (1 + b/a)$. Hence log $a + A = \log a + \log (1 + b/a) = \log \{a(1 + b/a)\} = \log (a + b)$.

In the table of subtraction logarithms (Table 4) each number in column D is the logarithm of some number n, and the corresponding number in column S is the logarithm of $n/(n-1)$. That is, if $D = \log n$, $S = \log n/(n - 1)$.

By means of this table, if we have log a and log b, where a and b are any given numbers, we can find log $(a - b)$ by taking the difference log a − log b, calling this difference D, looking out in column S the number corresponding to D, and subtracting this number from log a. In other words, if log a − log $b = D$, then log $a - S = \log (a - b)$.

The proof is as follows: If log a − log $b = D$, and $D = \log n$,

then $\log a - \log b = \log n$, or $\log (a/b) = \log n$, and $a/b = n$.
But

$$S = \log\left(\frac{n}{n-1}\right) = \log\left(\frac{\frac{a}{b}}{\frac{a}{b}-1}\right) = \log\frac{a}{a-b};$$

$$\therefore \log a - S = \log a - \log\left(\frac{a}{a-b}\right)$$
$$= \log a - \{\log a - \log (a-b)\}$$
$$= \log (a-b).$$

§ 2. Examples.

A few examples will make the matter clear.

Ex. 1. Suppose the legs of a right-angled triangle are 36.3 and 58.9, what is the hypothenuse?

$$\text{hyp.} = \sqrt{(58.9)^2 + (36.3)^2},$$

$\log (58.9)$	1.770	
$\log (58.9)^2$		$3.540 = \log a,$
$\log (36.3)$	1.560	
$\log (36.3)^2$		$3.120 = \log b,$
		$.420 = D = \log a - \log b,$
		$.140 = A,$
$\log (\text{hyp.})^2$		$3.680 = \log a + A = \log (a+b),$
$\log (\text{hyp.})$	1.840;	

$$\therefore \text{hyp.} = 69.2.$$

Ex. 2. Suppose the hypothenuse of a right-angled triangle is 69.2, and one of the sides is 58.9. What is the third side?

$$\text{Third side} = \sqrt{(69.2)^2 - (58.9)^2};$$

$\log (69.2)$	1.840	
$\log (69.2)^2$		$3.680 = \log a,$

log (58.9)	1.770	
log $(58.9)^2$		$3.540 = \log b,$
		$.140 = D = \log a - \log b,$
		$.560 = S,$
log $(\text{Third side})^2$		$3.120 = \log a - S = \log (a - b),$
log (Third side)	1.560;	
	$\therefore$ Third side = 36.3.	

Ex. 3. Suppose two sides of a spherical triangle are 37° and 62°, and the included angle is 50°.9. What is the third side?

The formula is

$$\cos (\text{side}) = \cos 37^\circ \cos 62^\circ + \sin 37^\circ \sin 62^\circ \cos 50^\circ.9;$$

cos 37°	9.902	
cos 62°	9.672	$9.574 = \log a;$
sin 37°	9.779	
sin 62°	9.946	
cos 50°.9	9.800	$9.525 = \log b,$
		$.049 = D = \log a - \log b,$
		$.277 = A,$
cos (side), 44°.8,		$9.851 = \log a + A = \log (a + b).$

Ex. 4. Suppose the three sides of a spherical triangle are 44°.8, 37° and 62°. What is the angle opposite the side 44°.8?

The formula is

$$\cos (\text{angle}) = \frac{\cos 44^\circ.8 - \cos 37^\circ \cos 62^\circ}{\sin 37^\circ \sin 62^\circ};$$

cos 44°.8		$9.851 = \log a,$
cos 37°	9.902	
cos 62°	9.672	$9.574 = \log b,$
		$.277 = D = \log a - \log b,$

$$.326 = S,$$

$$9.525 = \log a - S = \log (a - b);$$

sin 37°	9.779	
sin 62°	9.946	9.725
cos (angle), 50°.9,		9.800

If the reader will now work the last two examples in the old-fashioned way, the former by Napier's Analogies, and the latter by the standard formula for the case where the three sides of a spherical triangle are given, he can hardly fail to realize the vast superiority of Gaussian logarithms.

In the Tables, D, A, and S stand for "difference," "addition," and "subtraction." The initials A and S show not only which table is being used, but also whether the number taken from the table is to be added or subtracted. This addition or subtraction may be easily made by covering the intermediate numbers with a finger, a pencil, or a strip of paper.

In dealing with trigonometrical formulæ, the signs of the natural functions should be carefully noted. For example, take the formula

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

and suppose that $b = 100°$, $c = 120°$, and $A = 150°$. Then $\cos b$ and $\cos c$ are both minus, $\sin b$ and $\sin c$ are both plus, and $\cos A$ is minus. See Table, page 47. Hence the equation becomes

$$\cos a = \overset{-}{\cos}\, b \overset{-}{\cos}\, c + \overset{+}{\sin}\, b \overset{+}{\sin}\, c \overset{-}{\cos}\, A$$

$$\therefore \cos a = \cos b \cos c - \sin b \sin c \cos A.$$

Here subtraction logarithms are to be used. If it turns out that the term ($\sin b \sin c \cos A$) is numerically greater than the

term (cos b cos c), then the equation should be written

$$\cos a = -(\sin b \sin c \cos A - \cos b \cos c)$$

and the value of a will be greater than 90°, since cos a is minus.

§ 3. Advantages.

The advantages of Gaussian logarithms are:

(1) They do not require either the transformation of a formula to adapt it to logarithmic computation, or the use of an auxiliary angle for that purpose. At every step, the computer sees just what he is doing, and is not obliged to spend time in rearranging his data, or in computing intermediary functions which serve no purpose save to give a logarithmic shape to the work.

(2) They follow a general method, available for all cases, while the old-fashioned methods demand a special rearrangement of the formulæ for each particular case. With Gaussian logarithms, the numerous and complicated "forms of work" given in Bowditch (pages 257 *et seq.*) can nearly all be discarded in favor of a simple and direct solution of the original equation.

(3) They use only the functions (sines, cosines, etc.) of the original formula. These are frequently used again, before the end of the work is reached, and they do not have to be looked out in the tables a second time. Where auxiliary angles are employed, the sines, tangents, etc., of these angles have to be looked up, but are of no further use in the rest of the computation.

(4) They are more easily accommodated to the algebraic signs of the trigonometric functions, where the angles are larger than 90°.

(5) And lastly, and most important of all, they not only take less time, but give more accurate results than other methods. Many illustrations of this statement might be given, but for

present purposes it may be rested upon the general verdict of experienced computers.

In view of the obvious merits of Gaussian logarithms, it is singular that the nautical world does not seem yet to have taken them up. The astronomers have been employing them for many years. But navigators go on using complicated "forms of work," cumbrous tables of haversines, intricate and clumsy formulæ (such as Napier's Analogies) designed to give a logarithmic shape to other formulæ, and roundabout substitutions of auxiliary angles for the same purpose—when all the while a simple and direct method, free from all these complications, is available. However, it can scarcely be doubted that Gaussian logarithms will in time be used by all nautical workers—even by the British Navy. It took that well-known institution fourteen years to realize the merits of Lord Kelvin's compass,* and Lord Kelvin was a native Briton with a scientific reputation and prestige second to none. In the case of a product like Gaussian logarithms, handicapped by the label, "Made in Germany," it may take longer; but it seems certain that the invention of Gauss, like the Leibnitzian notation in calculus, will be universally adopted in the end.

* See "Life of Lord Kelvin," by S. P. Thompson, vol. 2, page 710.

CHAPTER II.

FIXING THE GREAT CIRCLE COURSE.

§ 4. Introductory Remarks.

(1) In the following pages we shall assume that the course of a ship at sea (which will be represented by C) is measured in accordance with the practice in the U. S. Navy, *i.e.*, from the north point easterly around the circle, from 0° to 360°; and we shall follow this system uniformly in the southern as well as in the northern hemisphere. Thus if the course is given as east 30° south, the angle $C = 120°$. If the course is stated to be south 50° west, the angle $C = 230°$.

(2) A vessel cannot at present be steered to any finer angle than entire degrees. Hence the course will be measured in degrees and tenths of a degree. This is the practice in the U. S. Navy, and it is ample for all practical purposes. Indeed, it is more than ample; for the Traverse Table in Bowditch, designed for practical navigation, does not attempt to subdivide degrees.

(3) North latitudes will be considered plus, and south latitudes minus. So if any increment of latitude ($d\varphi$) is northerly, it will be considered plus, and if southerly, minus. This conforms to the definition of C; for if C is less than 90° or greater than 270°, $d\varphi$ (which equals $ds \cos C$) is plus, since $\cos C$ is then plus; and if C is greater than 90° and less than 270°, $d\varphi$ is minus ($\cos C$ being then minus).

(4) Easterly departures (as given in the Traverse Table) are deemed plus, and westerly departures minus. This again conforms to the definition of C; for the departure $= ds \sin C$, and, accordingly, is plus where C is less than 180°, that is, where the departure is easterly, and minus where C is greater than 180°, that is, where the departure is westerly.

These conventional signs are not new to the student of coördinate geometry, who will recognize the departure as being the abscissa and the latitude difference as corresponding to the ordinate.

(5) The sign of the longitude follows the sign of the departure; for the longitude difference is the intercept made on the equator by meridians passing through the extremities of the departure, *i.e.*, long. diff. $= \frac{D}{\cos \varphi}$; and hence the sign of "long. diff." is determined by the sign of D ($\cos \varphi$ being always plus, since $\cos (-\varphi) = \cos \varphi$). As easterly departures are plus and westerly departures minus, so also easterly longitudes are plus and westerly longitudes minus. In both cases, the ship is the origin of measurement.

§ 5. To Find the Great Circle Initial Course from the Place of Departure to the Port of Destination.

There are various graphic methods of finding the initial great circle course. The objection to all such methods is that the measurements are dependent upon scales marked on paper, and the paper shrinks irregularly, and is variously affected by humidity, dryness, heat, and cold, so that the measurements are unreliable. Moreover, the minuteness of the distances on the paper makes them difficult to measure, even with the expenditure of much care and time in plotting; and conditions on shipboard are often unfavorable for plotting. Where special charts are available, they are apt to be large and unwieldy, and soon become confusing from the multiplicity of pencil lines, and of holes made by the points of the dividers. Graphic methods were sufficient in the days of sailing vessels; but in these modern days, when delay in the completion of voyages is measured in dollars per minute (see page 44), something more accurate is demanded by the up-to-date navigator. This greater degree of accuracy can be secured

only by calculation; and we shall confine our attention to methods of that nature.*

Let C be the position of the ship in latitude φ. Let A be the port of arrival in latitude φ_a. Let P be the north pole of the earth. Then the angle P is the difference of longitude of C and A, and the angle C is the initial course.

FIG. 1.

In any spherical triangle ABC,

$$\cot c \sin a = \cot C \sin B + \cos a \cos B.$$

Applying this formula to the triangle APC,

$$\cot PA \sin PC = \cot C \sin P + \cos PC \cos P,$$

$$\tan \varphi_a \cos \varphi = \cot C \sin P + \sin \varphi \cos P,$$

$$\cot C = \frac{\tan \varphi_a \cos \varphi - \sin \varphi \cos P}{\sin P} \qquad (1)$$

$$= \tan \varphi_a \cos \varphi \operatorname{cosec} P - \sin \varphi \cot P. \qquad (2)$$

This equation gives the course C in terms of φ_a, φ, and P, all of which may be either plus or minus. The signs of these

* In the Mathematical Laboratory of the University of Edinburgh, graphical methods have been almost altogether abandoned. *Nature*, Sept. 13, 1924, page 378.

quantities should be carefully noted. The latitudes φ_a and φ are plus for north latitudes and minus for south latitudes. The longitude difference P is plus when measured easterly from the vessel, and minus when measured westerly. See § 4, Rem. 5, page 9. Hence it is necessary, in using the equation, to bear in mind the following relations (see Table, page 47):

$$\tan(-\varphi_a) = -\tan\varphi_a,$$
$$\cos(-\varphi) = \cos\varphi,$$
$$\sin(-\varphi) = -\sin\varphi,$$
$$\operatorname{cosec}(-P) = -\operatorname{cosec} P,$$
$$\cot(-P) = -\cot P.$$

Or, generally, trig. function $(-$ angle$) = -$ trig. function (angle), except in the case of the cosine and its reciprocal the secant.

It is also necessary to use the relation $\cot C = \cot(180° + C)$ for all cases where the ship is sailing westerly; for here C is greater than 180°.

There are several ways of solving the equation. In no case is a minutely accurate result necessary; for an answer to tenths of a degree is sufficient for all practical purposes (see § 4, Rem. 2, page 8). This being so, and C being fortunately given by its cotangent, three place logarithms will suffice for all practical needs. See Table 2.

(1) The solution of the equation may be made by means of the natural trigonometrical functions to three decimal places. The necessary multiplications may be made with the aid of an extended multiplication table, such as Cotsworth's, or Crelle's, or better still by means of H. B. Hedrick's "Interpolation Tables." The last named work gives merely the figures that are necessary, omitting those beyond three places of decimals.

(2) The solution may be made by the agency of the azimuth tables, but with doubtful advantage. The azimuth tables are

prepared on different plans and for different purposes, and to utilize them for this purpose requires a transformation of the data, as well as a series of interpolations, which, in most cases, will take far more time than a direct multiplication under the first method above.

(3) The solution may, of course, be made by the usual logarithmic methods. For the application of logarithms, eq. (1) may be transformed as follows:

Factoring,

$$\cot C = \frac{\cos P \left(\frac{\tan \varphi_a \cos \varphi}{\cos P} - \sin \varphi \right)}{\sin P}$$

$$= \cot P \left(\frac{\tan \varphi_a \cos \varphi}{\cos P} - \sin \varphi \right).$$

Putting

$$\frac{\tan \varphi_a}{\cos P} = \tan \theta = \frac{\sin \theta}{\cos \theta},$$

$$\cot C = \cot P \left(\frac{\sin \theta}{\cos \theta} \cos \varphi - \sin \varphi \right)$$

$$= \frac{\cot P \sin (\theta - \varphi)}{\cos \theta};$$

$$\therefore \tan C = \frac{\tan P \cos \theta}{\sin (\theta - \varphi)}, \tag{3}$$

where

$$\tan \theta = \frac{\tan \varphi_a}{\cos P}. \tag{4}$$

Equations (3) and (4) give C.

(4) But it is believed that by far the best method is to use addition-subtraction logarithms. Not only is this method shorter, simpler, and more accurate than the others, but it is also an entirely general method, which, if the algebraic signs are properly noted, will lead mechanically to a correct result

without the necessity of paying any attention to the intermediate steps.

A few examples will make the whole matter clear.

§ 6. Examples.

Ex. 1. A ship is in the Atlantic. The noon observations have just fixed the position as lat. 33° N., long. 59° W. What is the initial great circle course to Queenstown (lat. 51°.8 N., long. 8°.3 W.)?

Here $P = +\ 50°.7$, and φ and φ_a are both plus. Hence the equation is

$$\cot C = \overset{+}{\tan}\ \varphi_a \overset{+}{\cos}\ \varphi \overset{+}{\operatorname{cosec}}\ P - \overset{+}{\sin}\ \varphi \overset{+}{\cot}\ P.$$

Subtraction logarithms are to be used.

tan φ_a,	51°.8,	.104	
cos φ,	33°,	9.924	
cosec P,	50°.7,	.111	10.139
sin φ,	33°,	9.736	
cot P,	50°.7,	9.913	9.649
			.490 D
			.170 S
cot C,	47°,		9.969

$C = 47° = \text{ans.}$

Ex. 2. What is the great circle initial course from a point off Japan, lat. 35°.5 N., long. 140°.9 E., to a point off Valparaiso, lat. 33° S., long. 71°.8 W.?

Here $P = +\ 147°.3$, φ is plus, and φ_a is minus. Hence cosec P is plus, cot P is minus, and tan φ_a is minus. Accordingly, the equation is

$$\cot C = \overset{-}{\tan}\ \varphi_a \overset{+}{\cos}\ \varphi \overset{+}{\operatorname{cosec}}\ P - \overset{+}{\sin}\ \varphi \overset{-}{\cot}\ P$$

or

$$-\cot C = \tan \varphi_a \cos \varphi \operatorname{cosec} P - \sin \varphi \cot P.$$

The signs being thus attended to, the rest of the work is merely numerical. Subtraction logarithms are used.

$\tan \varphi_a$, 33°,	9.813	
$\cos \varphi$, 35°.5,	9.911	
$\operatorname{cosec} P$, 147°.3,	.267	9.991
$\sin \varphi$, 35°.5,	9.764	
$\cot P$, 147°.3,	.192	9.956
		.035 D
		1.111 S
$-\cot C$, 85°.7,		8.880

$$C = 94°.3 = \text{ans.}$$

Ex. 3. Find the great circle initial course from San Francisco in lat. 37°.8 N., long. 122°.5 W., to Sydney in lat. 33°.9 S., long. 151°.2 E.

Here $P = -86°.3$, $\varphi = +37°.8$, $\varphi_a = -33°.9$. Hence the equation is

$$\cot C = \tan \varphi_a \cos \varphi \operatorname{cosec} P - \sin \varphi \cot P$$

or

$$\cot C = \tan \varphi_a \cos \varphi \operatorname{cosec} P + \sin \varphi \cot P.$$

Addition logarithms are used; and the signs being accounted for above, the rest of the work is merely numerical.

$\tan \varphi_a$, 33°.9,	9.827	
$\cos \varphi$, 37°.8,	9.898	
$\operatorname{cosec} P$, 86°.3,	.001	9.726
$\sin \varphi$, 37°.8,	9.787	
$\cot P$, 86°.3,	8.811	8.598
		1.128 D

		.031 *A*
cot *C*,	60°.25,	9.757

Since cot C = cot (180° + C), C is either 60°.25 or 240°.25. Here the course is westerly, and hence is greater than 180°, *i.e.*, it is 240°.25 = 240° 15′.

The above example is taken from Muir's Navigation (Ex. 66, page 283), where the exact answer is given as 240° 17′ 10″.

Ex. 4. Find the great circle initial course from Brisbane in lat. 27°.5 S., long. 153° E., to Acapulco, lat. 16°.8 N., long. 99°.9 W.

Here P = + 107°.1, φ = − 27°.5, φ_a = + 16°.8. The equation is

$$\cot C = \overset{+}{\tan}\ \varphi_a \overset{+}{\cos}\ \varphi \overset{+}{\operatorname{cosec}}\ P - \overset{-}{\sin}\ \varphi \overset{-}{\cot}\ P$$

or

$$\cot C = \tan \varphi_a \cos \varphi \operatorname{cosec} P - \sin \varphi \cot P.$$

The equation calls for subtraction logarithms.

tan φ_a,	16°.8,	9.480	
cos φ,	27°.5,	9.948	
cosec P,	107°.1,	.020	9.448
sin φ,	27°.5,	9.664	
cot P,	107°.1,	9.488	9.152
			.296 *D*
			.306 *S*
cot *C*,	82°.1,		9.142

C = 82°.1 = ans.

Ex. 5. Find the great circle initial course from Melbourne in lat. 37°.8 S., long. 145° E., to Callao, lat. 12°.1 S., long. 77°.1 W.

Here $P = + 137°.9$, $\varphi = - 37°.8$, $\varphi_a = - 12°.1$. The equation is

$$\cot C = \overset{-}{\tan}\ \varphi_a \overset{+}{\cos}\ \varphi \overset{+}{\operatorname{cosec}}\ P - \overset{-}{\sin}\ \varphi \overset{-}{\cot}\ P$$

or

$$- \cot C = \tan \varphi_a \cos \varphi \operatorname{cosec} P + \sin \varphi \cot P.$$

This equation calls for addition logarithms.

$\tan \varphi_a$, 12°.1,	9.331	
$\cos \varphi$, 37°.8,	9.898	
$\operatorname{cosec} P$, 137°.9,	.174	9.403
$\sin \varphi$, 37°.8,	9.787	
$\cot P$, 137°.9,	.044	9.831
		9.403
		.428 D
		.138 A
$-\cot C$, 47°.05,		9.969

$$C = 132°.95 = 132°\ 57'.$$

The above example is from Muir's Navigation (Ex. 65, page 283), where the exact answer is given as 132° 55′ 36″.

Ex. 6. Find the great circle initial course from a point off the southwest coast of Australia in lat. 35°.7 S., long. 118°.1 E., to a point off the Brazilian coast in lat. 22°.2 S., long. 41°.5 W.

Here $P = - 159°.6$, $\varphi = - 35°.7$, $\varphi_a = - 22°.2$.

$$\cot C = \overset{-}{\tan}\ \varphi_a \overset{+}{\cos}\ \varphi \overset{-}{\operatorname{cosec}}\ P - \overset{-}{\sin}\ \varphi \overset{+}{\cot}\ P$$
$$= \tan \varphi_a \cos \varphi \operatorname{cosec} P + \sin \varphi \cot P.$$

tan φ_a, 22°.2,	9.611	
cos φ, 35°.7,	9.910	
cosec P, 159°.6,	.458	9.979
sin φ, 35°.7,	9.766	
cot P, 159°.6,	.430	10.196
		9.979
		.217 D
		.206 A
cot C, 21°.6,		10.402

Since cot C = cot (180° + C), C is either 21°.6 or 201°.6. Here C is greater than 180°, and hence is 201°.6 = 201° 36′ (or S. 21° 36′ W.).

The above example is from Muir's Navigation (Ex. 60, page 270), where the exact answer is given as S. 21° 38′ W.

§ 7. To Find the Changes to be Made in the Great Circle Initial Course, in Order to Keep the Vessel on the Arc of a Great Circle. Exact Method.

The initial course being found as in the last section, the next question is how to correct this course from time to time, so as to keep the ship on the great circle path. Since the course of a vessel cannot be shaped to any finer angle than entire degrees (§ 4, Rem. 2, page 8), the navigator's only procedure is to maintain the initial course until the time comes to change it by one entire degree. Accordingly the question is, How far must the vessel steam, before changing the course 1°?

As already stated, we shall consider the course as measured from the north, easterly around the circle, from 0° to 360°.

Let P be the pole, and C the position of the ship in north

latitude φ, so that $PC = 90° - \varphi$. Let the angle C be the great circle initial course. Let A be the point at which this course is increased by one degree, so that $C' = C + 1°$. Let $CA = ds$ = distance sailed by the ship before this change in the course is made; and we desire to find ds.

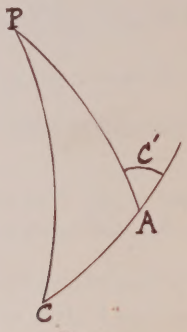

FIG. 2.

In any spherical triangle ABC,

$$\cot a \sin b = \cot A \sin C + \cos b \cos C.$$

Applying this formula to the triangle APC,

$$\cot PC \sin CA = \cot PAC \sin C + \cos CA \cos C,$$
$$\tan \varphi \sin ds = -\cot C' \sin C + \cos ds \cos C.$$

In this equation, all the quantities are known except ds. Dividing by $\cos C$,

$$\cot C' \tan C = \cos ds - \sin ds \frac{\tan \varphi}{\cos C}.$$

Putting

$$\frac{\tan \varphi}{\cos C} = \tan \theta = \frac{\sin \theta}{\cos \theta},$$

$$\cot C' \tan C = \cos ds - \sin ds \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\cos(ds + \theta)}{\cos\theta};$$

$$\therefore \cos(ds + \theta) = \cos\theta \tan C \cot C',$$

where $\tan\theta = \tan\varphi \sec C$.

These two equations determine ds; and the length of ds determines the point A at which the course is to be increased by one degree.

Ex. 1. A ship is in the North Atlantic (lat. 40° N.), sailing on a great circle course, N. 20° E. How far will the ship sail before the course is changed to N. 21° E.?

The work may be arranged as follows:

C' 21°			cot C' .415823
C 20°		sec C .027014	tan C 9.561066
φ 40°		tan φ 9.923814	
θ 41° 45′ 48″		tan θ 9.950828	cos θ 9.872682
$ds + \theta$ 44° 59′ 19″			cos $(ds + \theta)$ 9.849571
ds 3° 13′ 31″ = 193′.5			

§ 8. Approximate Method.

The foregoing method is exact. We now offer an approximate method, which is far briefer than the above, but is believed to be amply sufficient for all practical purposes.

Fig. 3.

Let C be the position of the ship in latitude φ, A the port of arrival in latitude φ_a, and P the pole.

$$\sin C = \frac{\sin PA \sin A}{\sin PC}$$
$$= \frac{\cos \varphi_a \sin A}{\cos \varphi}. \qquad (1)$$

In this equation, φ_a and A are constants. Differentiating, C and φ being the variables,

$$\cos C \frac{dC}{d\varphi} = \frac{\cos \varphi_a \sin A \sin \varphi}{\cos^2 \varphi}$$
$$= \frac{\cos \varphi_a \sin \varphi}{\cos \varphi} \cdot \frac{\sin A}{\cos \varphi}.$$

But from (1),

$$\frac{\sin A}{\cos \varphi} = \frac{\sin C}{\cos \varphi_a};$$
$$\therefore \cos C \frac{dC}{d\varphi} = \frac{\cos \varphi_a \sin \varphi}{\cos \varphi} \cdot \frac{\sin C}{\cos \varphi_a}$$
$$= \tan \varphi \sin C;$$
$$\therefore \frac{dC}{d\varphi} = \tan \varphi \tan C,$$
$$dC = d\varphi \tan \varphi \tan C. \qquad (2)$$

This equation is simple, symmetrical, and easy of application. In discussing it, the first point to be considered is the sign of dC. If ds be the distance in miles sailed over by the ship in a small interval of time, $d\varphi = ds \cos C$, and we have

$$dC = ds \cos C \tan \varphi \tan C$$
$$= ds \sin C \tan \varphi \qquad (3)$$
$$= D \tan \varphi, \qquad (4)$$

where D is the departure (of the Traverse Tables).

From this equation it appears that in northern latitudes,

where φ (and tan φ) is plus, dC will be plus in all cases of easterly departures, and minus in all cases of westerly departures. On the other hand, in southern latitudes, where φ (and hence tan φ) is minus, the converse will be the rule. The same result could be easily shown by diagrams. It can be remembered by noting that in north latitudes, for a ship sailing east, C is increasing, and dC is plus (just as easterly longitudes are plus, and easterly abscissas plus), while for a ship sailing west, the contrary is true. In southern latitudes, the sign of dC is opposite to what it is in north latitudes, *i.e.*, minus if sailing east and plus if sailing west.

Strictly speaking, eqs. (2) and (3) are true only for the infinitesimal unit of time at which the ship is in latitude φ, pursuing the course C. Practically, however, the equations may be taken as approximately true for longer intervals of time, such as 12 or even 24 hours. On this supposition, dC, ds, and $d\varphi$ are no longer infinitesimals, but represent the increments (which for dC and $d\varphi$ may be either positive or negative) taken on by the course, distance, and latitude in the interval of time assumed as the unit. On this theory, we should take, for C and φ, not the values of the course and latitude at the beginning of the unit time, nor yet the values at the end, but the mean or average values during this unit time interval. We may represent these mean values by C_m and φ_m. Eq. (3) then becomes

$$dC = ds \tan \varphi_m \sin C_m$$

from which

$$ds = \frac{dC}{\sin C_m} \cot \varphi_m.$$

In this equation, we know C_m, which is $(C + \frac{1}{2}°)$. We may write N for $dC/\sin C_m$, and may tabulate the values of N (which is done in Table 5), dC being expressed in miles (or minutes on the earth). We then have

$$ds = N \cot \varphi_m.$$

As φ is known, φ_m is obtained, as soon as φ' (the latitude at A) is calculated, for $\varphi_m = \frac{\varphi + \varphi'}{2}$. To get φ',

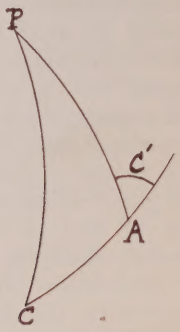

FIG. 4.

$$\frac{\sin PA}{\sin C} = \frac{\sin PC}{\sin PAC};$$

$$\therefore \cos \varphi' = \frac{\sin C}{\sin C'} \cos \varphi.$$

It will be convenient to write M for $\sin C/\sin C'$, and to tabulate the values of M, as in Table 5. Then the equations for determining ds are

$$\cos \varphi' = M \cos \varphi, \tag{5}$$

$$\varphi_m = \frac{\varphi + \varphi'}{2}, \tag{6}$$

$$ds = N \cot \varphi_m, \tag{7}$$

where M and N are taken from Table 5.

The above discussion relates to the case where a ship is in north latitude, sailing east. We shall now show that the results obtained are exactly the same for all other possible cases.

North latitude, sailing east:

dC is plus, φ and φ_m are plus, cot φ_m is plus, and $C < 180°$.

$$M = \frac{\sin C}{\sin (C + 1°)}, \quad \text{and} \quad \cos \varphi' = M \cos \varphi,$$

$$N = \frac{dC}{\sin (C + \frac{1}{2}°)}, \quad \text{and} \quad ds = N \cot \varphi_m.$$

North latitude, sailing west:

dC is minus, φ and φ_m are plus, cot φ_m is plus, and $C > 180°$.

$$M = \frac{-\sin C}{-\sin (C - 1°)} = \frac{\sin C}{\sin (C - 1°)}, \quad \text{and} \quad \cos \varphi' = M \cos \varphi,$$

$$N = \frac{-dC}{-\sin (C - \frac{1}{2}°)} = \frac{dC}{\sin (C - \frac{1}{2}°)}, \quad \text{and} \quad ds = N \cot \varphi_m.$$

South latitude, sailing east:

dC is minus, φ and φ_m are minus, cot φ_m is minus, and $C < 180°$.

$$M = \frac{\sin C}{\sin (C - 1°)}, \quad \text{and} \quad \cos \varphi' = M \cos \varphi,$$

$$N = \frac{-dC}{\sin (C - \frac{1}{2}°)}, \quad \text{and} \quad ds = N \cot \varphi_m.$$

South latitude, sailing west:

dC is plus, φ and φ_m are minus, cot φ_m is minus, and $C > 180°$.

$$M = \frac{-\sin C}{-\sin (C + 1°)} = \frac{\sin C}{\sin (C + 1°)}, \quad \text{and} \quad \cos \varphi' = M \cos \varphi,$$

$$N = \frac{dC}{-\sin (C + \frac{1}{2}°)}, \quad \text{and} \quad ds = N \cot \varphi_m.$$

Accordingly, it appears that in all cases the results for cos φ' and ds are the same, even (if our conventions are duly observed) in respect to the signs. This enables us to tabulate M and N in Table 5, and relieves the navigator from considering the minutiæ of individual cases. Whether sailing easterly or westerly, in north latitudes or south, all that he needs to do is to take his course, and read the table (Table 5) down for north latitudes and up for south latitudes. Opposite C will be found the values of log M and log N, and these will give φ' and ds from the equations cos $\varphi' = M$ cos φ, and $ds = N$ cot φ_m, which remain unaltered for all possible cases.

It may be noted in passing that in south latitudes, where φ_m and cot φ_m are minus, the result for ds will appear from the table to be minus; for the table does not show the sign of N (which in such a case is minus), and consequently the product N cot $\varphi_m = ds$, though really plus, appears to be negative. But this need not cause any difficulty; for ds is not regarded as having any algebraic sign, and we are interested only in its numerical value.

Eqs. (5), (6), and (7), in connection with Table 5, render the calculation of the corrections to the initial great circle course a very simple matter. For instance, take the example already worked on page 19. Table 5 gives (for a course of 20°) log M = 9.980, and log N = 2.234.

	M	9.980
40°	cos φ	9.884
43°	cos φ'	9.864
41°.5	cot φ_m	.053
	N	2.234
193′.5	ds	2.287

Thus with three-figure logarithms we get exactly the same result as with six-figure logarithms on page 19.

Taking as data the new course (21°) and the new latitude (43°), we may now repeat the work in the same way, and find how far the vessel will sail before the next change.

	M	9.981
43°	cos φ	9.864
45°.6	cos φ'	9.845
44°.3	cot φ_m	.011
	N	2.214
168′	ds	2.225

In other words, after sailing 168 miles on a course of 21°, the course will be increased to 22°. And so the process may be continued. But it should be noted that the point at which the course is changed, is wholly dependent upon the accuracy with which *ds* is logged off by the navigator; in other words, it is dead reckoning, and nothing more. As soon as new observations determine a new position with accuracy, the great circle initial course to the destination should be re-computed, and the process re-commenced *de novo;* for after the ship has once deviated from the former initial course, as in practice it always does more or less, the only practical procedure left for the navigator is to steer a new course from the place where the vessel actually is to the place for which it is bound.

The last named place may be the port of destination, or it may be some intermediate point, dependent upon ice, bodies of land, winds, currents, etc., according to the best judgment of the navigator.

As another example, let us suppose that a ship is in latitude 58° S., steaming S. 64° W., on a great circle course. What distance will she sail, before increasing the course to S. 65° W.?

The course here is 244°, and the latitude is south.

	M	9.996
58°	cos φ	9.724
58°.35	cos φ'	9.720
58°.17	cot φ_m	9.793
	N	1.823
41′.3	ds	1.616

Working this example at length, with six-place logarithms, we get $ds = 41'.3$, exactly the same as the above.

These two examples illustrate the surprising accuracy of the above method, although only three places of logarithms are used.

§ 9. Examples.

We continue below the examples given above on pages 13 *et seq.*

Ex. 1. The initial great circle course being 47°, the latitude + 33°, and the longitude − 59°, how far must the vessel steam before changing the course 1°? (Here dC is plus.)

	M	9.993
33°	cos φ	9.924
34°.3	cos φ'	9.917
33°.65	cot φ_m	.177
	N	1.911
122′	ds	2.088

Ex. 2. The initial great circle course is 94°, and the latitude is + 35°.5. How far must the ship sail before changing the course 1°? (The change is plus.)

	M	.001
35°.5	cos φ	9.911
35°.25	cos φ'	9.912
35°.4	cot φ_m	.148
	N	1.779
84′.5	ds	1.927

Ex. 3. The initial great circle course is 240°, and the latitude is 37°.8. How far must the vessel sail before changing the course 1°? (Here the increment dC is minus.)

	M	.004
37°.8	cos φ	9.898
37°.0	cos φ'	9.902
37°.4	cot φ_m	.117
	N	1.843
91′.2	ds	1.960

Ex. 4. The initial great circle course is 82°, and the latitude is − 27°.5. How far must the ship sail before changing the course 1°? (The increment dC is minus in this case.)

	M	.001
− 27°.5	cos φ	9.948
− 27°.2	cos φ'	9.949
− 27°.3	cot φ_m	.287
	N	1.783
117′.5	ds	2.070

Ex. 5. The initial great circle course is 133°, and the latitude is − 37°.8. Find how far the ship must sail before changing the course 1°. (Here dC is minus.)

	M	9.993
$-37°.8$	cos φ	9.898
$-38°.9$	cos φ'	9.891
$-38°.35$	cot φ_m	.102
	N	1.911
$103'$	ds	2.013

Ex. 6. The initial great circle course is 202°, and the latitude is $-35°.7$. Find how far the ship must sail before changing the course 1°. (The increment here is plus.)

	M	9.982
$-35°.7$	cos φ	9.910
$-38°.7$	cos φ'	9.892
$-37°.2$	cot φ_m	.120
	N	2.195
$206'.5$	ds	2.315

§ 10. To Find the Latitude and Longitude of the Points at which the Course is Changed.

As above shown, the great circle course is changed at intervals by an increase or decrease of one degree. The latitude of the point where the change is made, is found as a preliminary to determining ds. To find the longitude, take eq. (4), page 20, $dC = D \tan \varphi$. As dC here represents a finite, though small, quantity (1°), we write φ_m for φ.

$$\therefore dC = D \tan \varphi_m = \frac{D \sin \varphi_m}{\cos \varphi_m} = d\lambda \sin \varphi_m,$$

where $d\lambda = \dfrac{D}{\cos \varphi_m}$ = difference of longitude corresponding to

a departure D.

$$\therefore d\lambda = dC \operatorname{cosec} \varphi_m.$$

But dC is 1°, and hence $d\lambda$, measured in degrees, is equal to cosec φ_m.

As an illustration, take Ex. 1, page 26, where the initial great circle course is 47°, the latitude is + 33°, and the longitude is − 59°. What are the latitude and longitude of the point where the course is altered to 48°?

The latitude, obtained as an incident to finding ds, is $\varphi' = 34°.3$. The longitude increment, reckoned in degrees, is cosec φ_m, *i.e.*, the nat. cosec of 33°.6, or 1°.81. Hence long. $= -59° + 1°.81 = -57°.19$.

§ 11. Composite Sailing.

It often happens that a great circle track, if strictly followed, would take the ship into regions where navigation is dangerous from ice, or land forms, or other causes. In such a case, the navigator may decide not to go farther north (or south) than a given parallel of latitude (which may be called the limiting parallel). He should then shape his course on a great circle which, instead of cutting this parallel, merely touches it at a point of tangency. From this point he sails along the limiting parallel until he reaches the point of tangency corresponding to the great circle drawn from the destination to the limiting parallel, and this last named arc will conduct him to the port of arrival.

First then the navigator, after calculating the great circle initial course, will desire to know how far north (or south) it will take him. The point on this great circle which is nearest to the pole is called the vertex. To find the vertex, let C be the initial position of the ship, P the pole, and V the vertex. We know $PC = 90° - \varphi$, and the angle C is the initial course.

Then

$$\sin PV = \sin CP \sin C.$$

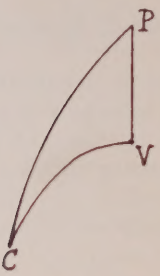

FIG. 5.

If φ_v be the latitude of V, this equation becomes

$$\cos \varphi_v = \cos \varphi \sin C. \tag{1}$$

To find the longitude of the vertex,

$$\cos CP = \cot C \cot P,$$
$$\cot P = \sin \varphi \tan C. \tag{2}$$

If φ_v takes the vessel too near the pole, it will be necessary to decide on a limiting latitude, which we may call φ_l. To find the point T at which a great circle from the starting point will just reach the limiting parallel without cutting it, change φ_v to φ_l in Eq. (1), and we have

$$\sin C = \sec \varphi \cos \varphi_l. \tag{3}$$

To find the longitude of T,

$$\cos P = \tan \varphi \cot \varphi_l. \tag{4}$$

Having thus fixed T, the navigator proceeds in the same way

to fix T', the point at which the great circle drawn from port of arrival becomes tangent to the limiting parallel. From T to T' he sails easterly (or westerly) on a small circle; for any great circle drawn to the destination from T, or from any point between T and T', would take the vessel above the limiting parallel. After reaching T', the navigator can resume a great circle path from T' to the destination.

CHAPTER III.

ADVANTAGES OF GREAT CIRCLE SAILING.

§ 12. Views of Certain Modern Writers.

According to the views of certain modern writers on navigation, it would seem that this chapter ought to be as brief as the famous chapter which was entitled "Snakes in Ireland," and which consisted merely of the title followed by a blank page.

Thus in a recent authoritative work (A. G. Mayor, Navigation, pages 48, 49) it is said: "To calculate a great circle course is a complex problem in spherical trigonometry," and "If, however, our ports of departure and of destination lie within 30° of the equator, there is practically no advantage in sailing a great circle course; and even on courses in fairly high latitudes, there is but little saving."

To test this statement, let us take for the port of departure, Brisbane (lat. 27° 28′ S., long. 153° 1′.6 E.), and for the port of destination, Coquimbo (lat. 29° 56′.2 S., long. 71° 21′ W.). By the formulæ in the next section,

Rhumb line distance	= 7177	miles.
Great circle "	= 6518	"
Saving	= 659	"

Evidently, therefore, to say that "there is practically no advantage in sailing a great circle course" in latitudes of less than 30°, is not altogether correct. The further statement that even "in fairly high latitudes, there is but little saving," is still more erroneous, as will be shown by the examples hereafter given.

In the excellent treatise on "Modern Navigation," by the

Rev. William Hall, page 123, it seems to be considered that about the only practical use of great circle sailing is to provide "a useful education in solving spherical triangles on the globe, as a preparation for astronomical work." On the same page it is stated that under latitude 60° and under a distance of 600 miles, the distances by Mercator and great circle sailing agree within one per cent.

This statement will not stand a practical test. Let us suppose that a vessel is in the Atlantic Ocean, lat. 58° N., long. 22° W., bound for Scapa Flow in the Orkneys, lat. 58° 58′ N., long. 3° 20′ W. Then by the accurate formulæ given in the next section, we get

Rhumb line distance	=	624.9 miles.
Great circle "	=	586.5 "
Difference	=	38.4 "

It thus appears that Mr. Hall's assertion that, under the conditions specified, the distances by Mercator and by great circle sailing agree within one per cent. is inaccurate; for in the above instance they differ by more than six times that estimate.

Perhaps the views of the above writers are due to a failure to take into account the accurate formula for the length of the loxodrome. Indeed, in a number of cases where the authorities give specific comparisons of the lengths of the great circle and of the rhumb line, the results are not in accordance with the rigorous formula. Some instances follow:

In Lecky, Wrinkles in Practical Navigation (19th ed.), page 660, the following example is given: Departure, Bergen, lat. 60° N., long. 5° E. Destination, Havana, lat. 23° N., long. 82° W.

Mercator distance	=	4340 miles.
Great circle "	=	4125 "
Difference	=	215 "

But the true Mercator distance is 4350 miles, making the difference 225 miles.

In W. C. Bergen, Epitome of Navigation (quoted in Lecky, Wrinkles in Practical Navigation (19th ed.), page 655), this example is given: Departure from a point near Tasmania, in lat. 40° S., long. 148° 30′ E., ship bound for Callao, lat. 12° 4′ S., long. 77° 14′ W.

Distance by Mercator track	=	7321 miles.
" " Great circle "	=	6772 "
Difference	=	549 "

By the correct formula the above figures are as follows:

Distance by Mercator track	=	7371.5 miles.
" " Great circle "	=	6777.5 "
Difference	=	594 "

In this connection, we find even a Cambridge don making a mistake. (We say this with fear and trembling.) Mr. Hugh Godfray's South Latitude Chart for Great Circle Sailing, in giving an illustrative example, states that 204 miles is the difference between the Mercator distance and the great circle distance from Cape Agulhas to Perth in Australia. But by the accurate formula the difference is 228.0 miles.

Finally, even Bowditch is not inerrant. (We utter this heresy not only with fear and trembling, but in a whisper.) In § 186, Bowditch asserts that the great circle distance from Sydney to Valparaiso is 748 miles shorter than the Mercator distance. The correct figures, by the accurate formula, are 783. Of a famous ancient savant it was said that it was "better to err with Aristarchus than to be right with the rest

of the world." There are probably many seafaring men who will say the same of Bowditch.*

In all the above cases, it will be observed that the saving which arises from following a great circle track is underestimated when tested by the exact formula for the length of the loxodrome.

Doubtless the reader will now demand to be introduced to the accurate formula, which we have referred to above. Accordingly, we give it below in a form which seems to be better adapted for practical calculation than the usual form. We also give a proof which, so far as we are aware, is new, and which appears at the same time to be as simple as the nature of the problem will allow.

§ 13. The Loxodrome.

The problem is: Given two points (on the earth) whose latitudes and longitudes are known, to find the length of the loxodrome connecting those points, and the constant angle which it makes with the meridians which it crosses.

The loxodrome may be most conveniently considered as commencing near the north pole (which, however, it never actually attains) and running southeast or southwest spirally around the earth until it approaches the south pole. Its distance from the north pole is the complement of the geographical (not geocentric) latitude, and is an increasing func-

* In justice to the eminent mathematicians who have successively revised Bowditch, it should, perhaps, be added that in arriving at the above figures they probably did not aim at minute accuracy, but only at a practical and approximate result. To get the figures stated in the text, it is necessary to use seven-place logarithms, and to carry the calculations to 100ths of a second. This, of course, is not a practical procedure for navigators. However, when the question is thus minutely examined, there is found to be far more advantage in the great circle method than would appear from the figures in Bowditch. The point is of importance only as indicating the frequency with which the advantages of great circle sailing are underestimated.

tion from pole to pole. We shall call it δ. The constant angle which the loxodrome makes with each meridian will be considered as measured from the north, either easterly or westerly, around to the portion of the curve already traced out by the moving point (*i.e.*, the portion of the curve behind the moving point and to the northward of it). The angle thus measured does not exceed 90°.

Let the annexed figure represent a section of the earth through a given point C on its surface and the axis OP, O being

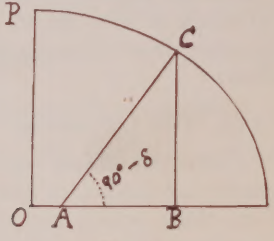

FIG. 6.

the center of the earth and P the north pole. This section is an ellipse, whose semi-axes are 6378.388 and 6356.909 kilometers respectively, and whose eccentricity $= e = .082$. (See Amer. Naut. Alm. (1925), page viii). If the coördinates of C are x and y, the subnormal $AB = x(1 - e^2)$. Hence tan CAB $= \cot \delta = \dfrac{y}{x(1 - e^2)} = \dfrac{a^2 y}{b^2 x}$.

$$\therefore \frac{y}{x} = \frac{b^2}{a^2} \cot \delta = \frac{b^2}{a^2} \frac{\cos \delta}{\sin \delta},$$

$$\frac{\frac{y}{b}}{b \cos \delta} = \frac{\frac{x}{a}}{a \sin \delta},$$

$$\frac{\left(\frac{y}{b}\right)^2}{b^2 \cos^2 \delta} = \frac{\left(\frac{x}{a}\right)^2}{a^2 \sin^2 \delta} = \frac{1}{a^2 \sin^2 \delta + b^2 \cos^2 \delta};$$

$$\therefore y^2 = \frac{b^4 \cos^2 \delta}{a^2 \sin^2 \delta + b^2 \cos^2 \delta} = \frac{\frac{b^4}{a^2} \cos^2 \delta}{1 - e^2 \cos^2 \delta}. \qquad (1)$$

Similarly,

$$x^2 = \frac{a^4 \sin^2 \delta}{a^2 \sin^2 \delta + b^2 \cos^2 \delta} = \frac{a^2 \sin^2 \delta}{1 - e^2 \cos^2 \delta}. \qquad (2)$$

Substituting these values of y^2 and x^2 in the formula for the radius of curvature of the ellipse,

$$\rho = \frac{(a^4 y^2 + b^4 x^2)^{3/2}}{a^4 b^4},$$

we get

$$\begin{aligned} \rho &= \frac{1}{a^4 b^4} \left(\frac{a^2 b^4 \cos^2 \delta}{1 - e^2 \cos^2 \delta} + \frac{a^2 b^4 \sin^2 \delta}{1 - e^2 \cos^2 \delta} \right)^{3/2} \\ &= \frac{b^2}{a} \left(\frac{1}{1 - e^2 \cos^2 \delta} \right)^{3/2} \\ &= \frac{a(1 - e^2)}{(1 - e^2 \cos^2 \delta)^{3/2}}. \end{aligned} \qquad (3)$$

Now let BCD be the curve of the loxodrome, making a constant angle $PCB = c$ with the meridian. As PC revolves

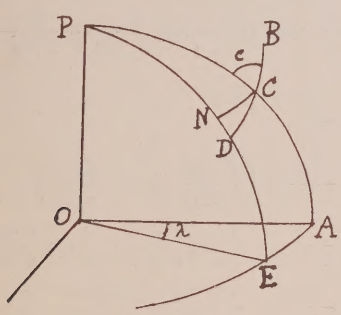

FIG. 7.

into the position PN, the arc CN, which has x for its radius, becomes $CN = x\,d\lambda$, where λ = the angle AOE = difference of longitude of C and N. The arc $ND = \rho\,d\delta$, where ρ = radius of curvature of ellipse at C.

$$\therefore \frac{CN}{ND} = \cot NCD = \tan PCB = \tan c;$$

$$\therefore \frac{x\,d\lambda}{\rho\,d\delta} = \tan c,$$

$$d\lambda = \frac{\tan c \cdot \rho \cdot d\delta}{x}.$$

Substituting the values of ρ and x from eqs. (3) and (2), and reducing,

$$d\lambda = \frac{\tan c(1 - e^2)d\delta}{\sin \delta(1 - e^2 \cos^2 \delta)};$$

$$\therefore \lambda = \tan c(1 - e^2) \int \frac{d\delta}{\sin \delta(1 - e^2 \cos^2 \delta)}.$$

To integrate this expression, put $e \cos \delta = z$, from which $-e \sin \delta\, d\delta = dz$, $d\delta = -\frac{dz}{e \sin \delta}$, and $\sin^2 \delta = \frac{e^2 - z^2}{e^2}$.

$$\therefore \lambda = -\tan c(1 - e^2)e \int \frac{dz}{(e^2 - z^2)(1 - z^2)}.$$

But

$$\int \frac{dz}{(e^2 - z^2)(1 - z^2)}$$

$$= \frac{1}{1 - e^2} \int \left(\frac{dz}{e^2 - z^2} - \frac{dz}{1 - z^2} \right)$$

$$= \frac{1}{1 - e^2} \left(\frac{1}{2e} \log \frac{e + z}{e - z} - \frac{1}{2} \log \frac{1 + z}{1 - z} \right)$$

$$= \frac{1}{1 - e^2} \left(\frac{1}{2e} \log \frac{e + e \cos \delta}{e - e \cos \delta} - \frac{1}{2} \log \frac{1 + e \cos \delta}{1 - e \cos \delta} \right)$$

$$= \frac{1}{1-e^2}\left(\frac{1}{2e}\log\frac{1+\cos\delta}{1-\cos\delta} - \frac{1}{2}\log\frac{1+\cos\omega}{1-\cos\omega}\right)$$

where we put $e\cos\delta = \cos\omega$,

$$= \frac{1}{1-e^2}\left(\frac{1}{2e}\log\cot^2\frac{\delta}{2} - \frac{1}{2}\log\cot^2\frac{\omega}{2}\right)$$

$$= \frac{1}{1-e^2}\left(\frac{1}{e}\log\cot\frac{\delta}{2} - \log\cot\frac{\omega}{2}\right);$$

$$\therefore \lambda = -\tan c\cdot e\left(\frac{1}{e}\log\cot\frac{\delta}{2} - \log\cot\frac{\omega}{2}\right)$$

$$= \tan c\left(e\log\cot\frac{\omega}{2} - \log\cot\frac{\delta}{2}\right).$$

Taking this integral between the limits δ and δ' (the corresponding longitudes being λ and λ', and the accented quantities representing the more southerly point, so that δ' is larger than δ), we get

$$\lambda' - \lambda = \tan c\left(e\log\cot\frac{\omega'}{2} - \log\cot\frac{\delta'}{2} - e\log\cot\frac{\omega}{2} + \log\cot\frac{\delta}{2}\right)$$

$$= \tan c\left(\log\cot\frac{\delta}{2} - \log\cot\frac{\delta'}{2} - e\left(\log\cot\frac{\omega}{2} - \log\cot\frac{\omega'}{2}\right)\right);$$

$$\therefore \cot c = \frac{\log\cot\frac{\delta}{2} - \log\cot\frac{\delta'}{2} - e\left(\log\cot\frac{\omega}{2} - \log\cot\frac{\omega'}{2}\right)}{\lambda' - \lambda}.$$

The logarithms in the above expression are Naperian. They may be converted into common logarithms by multiplying both terms of the fraction by $M = .4342945$, which

gives

$$\cot c = \frac{\log \cot\frac{\delta}{2} - \log \cot\frac{\delta'}{2} - e\left(\log \cot\frac{\omega}{2} - \log \cot\frac{\omega'}{2}\right)}{M(\lambda' - \lambda)}$$

and the logarithms are now common logarithms.

The angle c being thus determined, the length of the loxodrome follows at once; for $CD = \frac{ND}{\sin NCD} = \frac{ND}{\cos c}$;

$$\therefore ds = \frac{d\delta}{\cos c};$$

$$\therefore s = \int_{\delta}^{\delta'} \frac{d\delta}{\cos c} = \frac{\delta' - \delta}{\cos c}.$$

For convenience of reference, we catalogue once more the quantities in the above formulæ.

c is the constant angle which the loxodrome makes with the meridian. It is measured from the north to the portion of the curve already traced out, and is always less than 90°.

$M = .4342945$, and $\log M = 9.6377843$.

$\lambda' - \lambda$ is the difference of longitude of the two places in question, and may be always considered as plus, since the direction in which longitude is measured is purely arbitrary.

δ is the polar distance of the more northerly place, and δ' of the more southerly.

$e = .082$, and $\log e = 8.9138042$ (see Amer. Naut. Alm. (1925), page viii).

ω and ω' are auxiliary quantities ($\cos \omega = e \cos \delta$, and $\cos \omega' = e \cos \delta'$).

The logarithms used are common logarithms.

As an example, we take the following from Bowditch (page 81):

Find the loxodromic course and distance from lat. 40° N., long. 70° W., to lat. 30° S., long. 10° W.

Here $\delta' = 120°$, $\delta = 50°$, $\delta' - \delta = 70° = 4200'$, $\lambda' - \lambda = 60° = 1.0472$ (in radian measure).

$\cos \omega = e \cos \delta$		$\cos \omega' = e \cos \delta'$	
e	8.91380	e	8.91380
$\cos \delta$, 50°,	9.80807	$\cos \delta'$, 120°,	9.69897 (n)
$\cos \omega$	8.72187	$\cos \omega'$	8.61277 (n)
ω	86° 58′.7	ω'	92° 21′
$\frac{\omega}{2}$	43° 29′.3	$\frac{\omega'}{2}$	46° 10′.5
$\cot \frac{\delta}{2}$, 25°,	10.33133	$\cot \frac{\omega}{2}$, 43° 29′.3,	10.02292
$\cot \frac{\delta'}{2}$, 60°,	9.76144	$\cot \frac{\omega'}{2}$, 46° 10′.5,	9.98218
	.56989		.04074
	.00334 = .082 × .04074		
	.56655	M	9.63778
log (.56655)	9.75324	$\lambda' - \lambda$	.02003
$M(\lambda' - \lambda)$	9.65781	$M(\lambda' - \lambda)$	9.65781

$\cot c$ .09543

$c = 38° 45'.3$

$s = \dfrac{\delta' - \delta}{\cos c}$

$\delta' - \delta$, 4200′,	3.62325
$\cos c$, 38° 45′.3,	9.89200
s, 5386′,	3.73125

$s = 5386$ miles = ans.

As another example, we may take the following: Find the loxodrome course and distance from Nantucket Shoals Lightship (lat. 40° 37′.1 N., long. 69° 36′.5 W.) to Liverpool (lat. 53° 26′.6 N., long. 3° 2′.5 W.). (This is the track from New York to Liverpool, after the vessel has cleared Long Island.)

$\delta' = 49° 22'.9$, $\delta = 36° 33'.4$, $\delta' - \delta = 12° 49'.5 = 769'.5$, $\lambda' - \lambda = 66° 34' = 1.16181$ (rad. meas.).

$\cos \omega = e \cos \delta$			$\cos \omega' = e \cos \delta'$		
e	8.91380		e	8.91380	
$\cos \delta$	9.90486		$\cos \delta'$	9.81359	
$\cos \omega$	8.81866		$\cos \omega'$	8.72739	
ω	86° 13′.4		ω'	86° 56′.4	
$\frac{\omega}{2}$	43° 6′.7		$\frac{\omega'}{2}$	43° 28′.2	
$\cot \frac{\delta}{2}$, 18° 16′.7,	.48110		$\cot \frac{\omega}{2}$, 43° 6′.7,	.02864	
$\cot \frac{\delta'}{2}$, 24° 41′.4,	.33749		$\cot \frac{\omega'}{2}$, 43° 28′.2,	.02321	
	.14361			.00543	
	.00045	$= .082 \times .00543$			
	.14316		M	9.63778	
log (.14316)	9.15582		$\lambda' - \lambda$	.06514	
$M(\lambda' - \lambda)$	9.70292		$M(\lambda' - \lambda)$	9.70292	

$\cot c$ 9.45290

$c = 74° 9'.6$

$s = \dfrac{\delta' - \delta}{\cos c}$

$$\begin{aligned} \delta' - \delta,\ 769.5, &\quad 2.88621 \\ \cos c,\ 74^\circ\ 9'.6, &\quad 9.43609 \\ \hline s,\ 2819, &\quad 3.45012 \end{aligned}$$

$s = 2819$ miles = ans.

To compare this result with the great circle distance, we find the latter from the equation

$$\cos (\text{dist.}) = \sin \varphi \sin \varphi_a + \cos \varphi \cos \varphi_a \cos d\lambda$$

where φ and φ_a are the latitudes of the two places, and $d\lambda$ their difference of longitude.

This equation gives $2721'$ for the great circle distance between the points above specified. Hence we have

Loxodrome distance	=	2819 miles.
Great circle "	=	2721 "
Saving	=	98 "

§ 14. Illustrative Example.

Modern navigation is conducted in large ships, and the tendency is for them to become larger. Millions of dollars are locked up in ship and cargo. The daily expenditures for fuel, for stores of various kinds, and for the wages of the small army of employees required to man the ship, are enormous. While the voyage is going on, the capital is idle, while the expenses are heavy and continuous. Hence the demand is unceasing for any facility that will shorten the duration of voyages. A navigating officer who can bring his vessel to port even a few hours ahead of the average time of the voyage, effects a substantial economy; and when this is multiplied by the number of voyages per year, it becomes a total which cannot be ignored.

As a concrete example, showing the upper limit of pecuniary saving that can be secured by great circle navigation, let us take a vessel in the highest class of marine architecture, such

as the Leviathan or the Majestic. The former ship is 950 feet long, 100 feet wide, and has a gross tonnage of 59,957. The value is about $15,000,000. She carries 3210 passengers, and is manned by a crew of 1200. She consumes 900 tons of oil per day, costing $10,989. The total running expenses per day are about $20,000. She makes 12 round trips (or 24 single crossings) per year, at an average speed of 24½ knots.

In figuring the annual interest return that should be derived from the capital invested, it is necessary to allow not only 6 per cent. on the investment, but enough for repairs, alterations, and replacements, and for the risk of total loss (which is covered by insurance). It is believed that 15 per cent. is conservative. Hence the capital of $15,000,000 represents a daily interest charge of $6164.

The value of the freight carried by an ocean steamer is a matter of guesswork. In general, perhaps the value of the freight is about equal to the value of the ship itself—in this case, about 15 million dollars. The daily interest charge on this sum, at 6 per cent., comes to $2465.

Of the 3210 passengers, no doubt at least one third (or 1070) are adult males. At the present time, when carpenters and plumbers get $10 and $12 per day, and plasterers $18 per day, it is safe to assume that the value of the time of these 1070 passengers does not average less than $20 per day, or $21,400 in all.

Thus we have the following schedule of the amount that would be saved by the saving of one day in the time of the voyage of a steamer of the first class:

One day's interest charge on vessel,	$ 6,164.
" " " " " freight,	2,465.
Value of time of adult male passengers,	21,400.
Daily operating expenses of ship,	20,000.
Cost of each day's sailing,	$50,029.

This sum is $2084 per hour, or $35 per minute.

In the books of account of the owners, the first and last items are the only ones entered. These aggregate $26,164 per day, or $1090 per hour, or $18 per minute.

It is shown above (page 43) that the great circle track from New York to Liverpool is 98 miles shorter than the loxodromic track. At the rate of the Leviathan (24½ knots) this amounts to a saving of exactly four hours, or one sixth of a day; and in money this saving is one sixth of $50,029, or $8338 per trip. As the vessel makes 24 trips per year, the total annual saving is $200,112. From the point of view of the owners, the saving is $104,640 per year.

The foregoing figures are, it is true, only approximate, but they certainly indicate a tremendous economy. In respect to the item of fuel, the figures are pretty nearly exact. One sixth of a day's consumption of oil on each trip is $1831.50, and in a year (24 trips) this comes to $43,956. Hence the great circle track saves the Leviathan $43,956 per year in fuel alone.*

* To keep the ship on the arc of a great circle the navigator must know his position accurately at all times. Where delay means a loss of $35 per minute, it is imperative that he should endeavor to secure the utmost attainable accuracy both in taking his observations, and in working them up. The chief difficulty arises, as has always been the case, in the calculations for the longitude. The ordinary time sight depends upon dead reckoning, and is liable to be very fallacious. The only alternative method (Sumner Lines of Position) contains an inherent defect which precludes it from ever giving the best results. In view of these facts, the author ventures to call attention to a method of finding latitude and longitude, published recently by him, which is not dependent upon dead reckoning, is free from the intrinsic defects of Sumner lines, and aims to secure the maximum accuracy of which the observations are capable. At the same time it is believed to be shorter than any other method. See North Star Navigation, by L. M. Berkeley, N. Y., 1924. The method consists, briefly, in a new process for working up simultaneous observations of the North Star and of some other star. As the title indicates, it is available only in the northern hemisphere; but this hemisphere contains 75 per cent. of the land on the earth, 93 per cent. of the population of the earth, and probably 99 per cent. of the world's commerce and navigation. Hence it is believed

The above estimates relate to vessels of the largest size. For smaller ships perhaps an estimate can be made in a rough way, by taking the ratio of their tonnage to the tonnage of the Leviathan; but for any individual case, similar and more exact figures can be easily obtained.

§ 15. Conclusion.

In conclusion we venture to repeat what has been said before, that great circle sailing has attained in modern times an importance never dreamed of in the days of sailing vessels. In these modern days of speed, manifested in airplanes, railway cars, motor cars, motor boats, and steamships, it behooves the navigator, as never before, to trace out and follow the great circle path. To this end, we have endeavored above to give a method which requires only a few minutes' figuring, and is as accurate as the present facilities for steering require. It is believed that the method is sufficiently exact for practical purposes and sufficiently brief for the practical man. It is now submitted to the public, in the hope that, to those who have arrived thus far, and are about to close the book, it may not seem that the time necessary to learn the method has been wasted.

that the method may fairly claim to be a general one, of interest to all navigators, and especially to those who are ambitious to secure the immense economies obtainable from great circle sailing.

APPENDIX OF TABLES.

Explanation of Tables.

Table 1 is a table of common logarithms, arranged in columns parallel with the corresponding numbers.

Table 2 is a table of the logarithms of the trigonometric functions, to tenths of a degree.

Table 3 is a table of addition logarithms. For the construction and use, see page 2.

Table 4 is a table of subtraction logarithms. For the construction and use, see page 2.

Table 5 gives M and N. See pages 21 *et seq.* Where the ship is in northern latitudes, the course (C) is read down the page. Where the ship is in southern latitudes, C is read up the page. In both cases, C runs from 0° to 360°.

The following scheme is of frequent use in trigonometric work:

	$-A$	$90° - A$	$90° + A$	$180° - A$	$180° + A$	$270° - A$	$270° + A$	$360° - A$
sin	$-\sin A$	$\cos A$	$\cos A$	$\sin A$	$-\sin A$	$-\cos A$	$-\cos A$	$-\sin A$
cos	$\cos A$	$\sin A$	$-\sin A$	$-\cos A$	$-\cos A$	$-\sin A$	$\sin A$	$\cos A$
tan	$-\tan A$	$\cot A$	$-\cot A$	$-\tan A$	$\tan A$	$\cot A$	$-\cot A$	$-\tan A$
cot	$-\cot A$	$\tan A$	$-\tan A$	$-\cot A$	$\cot A$	$\tan A$	$-\tan A$	$-\cot A$
sec	$\sec A$	$\operatorname{cosec} A$	$-\operatorname{cosec} A$	$-\sec A$	$-\sec A$	$-\operatorname{cosec} A$	$\operatorname{cosec} A$	$\sec A$
cosec	$-\operatorname{cosec} A$	$\sec A$	$\sec A$	$\operatorname{cosec} A$	$-\operatorname{cosec} A$	$-\sec A$	$-\sec A$	$-\operatorname{cosec} A$

The whole of the above table (except the signs) can be carried in memory by noting that the sin, cos, tan, etc., of $(180° \pm A)$ or of $(360° \pm A)$ are the same functions of A,

while the sin, cos, tan, etc., of $(90° \pm A)$ or of $(270° \pm A)$ are the complementary functions of A. As to the signs, they are best remembered from the values of tan, cot, etc., in terms of the sine and cosine, the sine being positive in the first two quadrants and negative in the last two, and the cosine being positive in the first and fourth quadrants and negative in the other two. Thus $\cot A = \frac{\cos A}{\sin A}$, and if A is in the fourth quadrant, this is plus divided by minus, *i.e.*, minus.

TABLE 1.

Common Logarithms.

N	Log.	N	Log.	N	Log.	N	Log.	N	Log.	N	Log.	N	Log.
100	0.000	140	0.146	180	0.255	220	0.342	260	0.415	300	0.477	340	0.531
101	.004	141	.149	181	.258	221	.344	261	.417	301	.479	341	.533
102	.009	142	.152	182	.260	222	.346	262	.418	302	.480	342	.534
103	.013	143	.155	183	.262	223	.348	263	.420	303	.481	343	.535
104	.017	144	.158	184	.265	224	.350	264	.422	304	.483	344	.537
105	0.021	145	0.161	185	0.267	225	0.352	265	0.423	305	0.484	345	0.538
106	.025	146	.164	186	.270	226	.354	266	.425	306	.486	346	.539
107	.029	147	.167	187	.272	227	.356	267	.427	307	.487	347	.540
108	.033	148	.170	188	.274	228	.358	268	.428	308	.489	348	.542
109	.037	149	.173	189	.276	229	.360	269	.430	309	.490	349	.543
110	0.041	150	0.176	190	0.279	230	0.362	270	0.431	310	0.491	350	0.544
111	.045	151	.179	191	.281	231	.364	271	.433	311	.493	351	.545
112	.049	152	.182	192	.283	232	.365	272	.435	312	.494	352	.547
113	.053	153	.185	193	.286	233	.367	273	.436	313	.496	353	.548
114	.057	154	.188	194	.288	234	.369	274	.438	314	.497	354	.549
115	0.061	155	0.190	195	0.290	235	0.371	275	0.439	315	0.498	355	0.550
116	.064	156	.193	196	.292	236	.373	276	.441	316	.500	356	.551
117	.068	157	.196	197	.294	237	.375	277	.442	317	.501	357	.553
118	.072	158	.199	198	.297	238	.377	278	.444	318	.502	358	.554
119	.076	159	.201	199	.299	239	.378	279	.446	319	.504	359	.555
120	0.079	160	0.204	200	0.301	240	0.380	280	0.447	320	0.505	360	0.556
121	.083	161	.207	201	.303	241	.382	281	.449	321	.507	361	.558
122	.086	162	.210	202	.305	242	.384	282	.450	322	.508	362	.559
123	.090	163	.212	203	.307	243	.386	283	.452	323	.509	363	.560
124	.093	164	.215	204	.310	244	.387	284	.453	324	.511	364	.561
125	0.097	165	0.217	205	0.312	245	0.389	285	0.455	325	0.512	365	0.562
126	.100	166	.220	206	.314	246	.391	286	.456	326	.513	366	.563
127	.104	167	.223	207	.316	247	.393	287	.458	327	.515	367	.565
128	.107	168	.225	208	.318	248	.394	288	.459	328	.516	368	.566
129	.111	169	.228	209	.320	249	.396	289	.461	329	.517	369	.567
130	0.114	170	0.230	210	0.322	250	0.398	290	0.462	330	0.519	370	0.568
131	.117	171	.233	211	.324	251	.400	291	.464	331	.520	371	.569
132	.121	172	.236	212	.326	252	.401	292	.465	332	.521	372	.571
133	.124	173	.238	213	.328	253	.403	293	.467	333	.522	373	.572
134	.127	174	.241	214	.330	254	.405	294	.468	334	.524	374	.573
135	0.130	175	0.243	215	0.332	255	0.407	295	0.470	335	0.525	375	0.574
136	.134	176	.246	216	.334	256	.408	296	.471	336	.526	376	.575
137	.137	177	.248	217	.336	257	.410	297	.473	337	.528	377	.576
138	.140	178	.250	218	.338	258	.412	298	.474	338	.529	378	.577
139	.143	179	.253	219	.340	259	.413	299	.476	339	.530	379	.579

TABLE I (*Continued*).
COMMON LOGARITHMS.

N	Log.	*N*	Log.	*N*	Log.	*N*	Log.	*N*	Log.	*N*	Log.	*N*	Log.
380	0.580	420	0.623	460	0.663	500	0.699	540	0.732	580	0.763	620	0.792
381	.581	421	.624	461	.664	501	.700	541	.733	581	.764	621	.793
382	.582	422	.625	462	.665	502	.701	542	.734	582	.765	622	.794
383	.583	423	.626	463	.666	503	.702	543	.735	583	.766	623	.794
384	.584	424	.627	464	.667	504	.702	544	.736	584	.766	624	.795
385	0.585	425	0.628	465	0.667	505	0.703	545	0.736	585	0.767	625	0.796
386	.587	426	.629	466	.668	506	.704	546	.737	586	.768	626	.797
387	.588	427	.630	467	.669	507	.705	547	.738	587	.769	627	.797
388	.589	428	.631	468	.670	508	.706	548	.739	588	.769	628	.798
389	.590	429	.632	469	.671	509	.707	549	.740	589	.770	629	.799
390	0.591	430	0.633	470	0.672	510	0.708	550	0.740	590	0.771	630	0.799
391	.592	431	.634	471	.673	511	.708	551	.741	591	.772	631	.800
392	.593	432	.635	472	.674	512	.709	552	.742	592	.772	632	.801
393	.594	433	.636	473	.675	513	.710	553	.743	593	.773	633	.801
394	.595	434	.637	474	.676	514	.711	554	.744	594	.774	634	.802
395	0.597	435	0.638	475	0.677	515	0.712	555	0.744	595	0.775	635	0.803
396	.598	436	.639	476	.678	516	.713	556	.745	596	.775	636	.803
397	.599	437	.640	477	.679	517	.713	557	.746	597	.776	637	.804
398	.600	438	.641	478	.679	518	.714	558	.747	598	.777	638	.805
399	.601	439	.642	479	.680	519	.715	559	.747	599	.777	639	.806
400	0.602	440	0.643	480	0.681	520	0.716	560	0.748	600	0.778	640	0.806
401	.603	441	.644	481	.682	521	.717	561	.749	601	.779	641	.807
402	.604	442	.645	482	.683	522	.718	562	.750	602	.780	642	.808
403	.605	443	.646	483	.684	523	.719	563	.751	603	.780	643	.808
404	.606	444	.647	484	.685	524	.719	564	.751	604	.781	644	.809
405	0.607	445	0.648	485	0.686	525	0.720	565	0.752	605	0.782	645	0.810
406	.609	446	.649	486	.687	526	.721	566	.753	606	.782	646	.810
407	.610	447	.650	487	.688	527	.722	567	.754	607	.783	647	.811
408	.611	448	.651	488	.688	528	.723	568	.754	608	.784	648	.812
409	.612	449	.652	489	.689	529	.723	569	.755	609	.785	649	.812
410	0.613	450	0.653	490	0.690	530	0.724	570	0.756	610	0.785	650	0.813
411	.614	451	.654	491	.691	531	.725	571	.757	611	.786	651	.814
412	.615	452	.655	492	.692	532	.726	572	.757	612	.787	652	.814
413	.616	453	.656	493	.693	533	.727	573	.758	613	.787	653	.815
414	.617	454	.657	494	.694	534	.728	574	.759	614	.788	654	.816
415	0.618	455	0.658	495	0.695	535	0.728	575	0.760	615	0.789	655	0.816
416	.619	456	.659	496	.695	536	.729	576	.760	616	.790	656	.817
417	.620	457	.660	497	.696	537	.730	577	.761	617	.790	657	.818
418	.621	458	.661	498	.697	538	.731	578	.762	618	.791	658	.818
419	.622	459	.662	499	.698	539	.732	579	.763	619	.792	659	.819

TABLE I (*Continued*).
COMMON LOGARITHMS.

N	Log.	N	Log.	N	Log.	N	Log.	N	Log.	N	Log.	N	Log.
660	0.820	700	0.845	740	0.869	780	0.892	820	0.914	860	0.934	900	0.954
661	.820	701	.846	741	.870	781	.893	821	.914	861	.935	901	.955
662	.821	702	.846	742	.870	782	.893	822	.915	862	.936	902	.955
663	.822	703	.847	743	.871	783	.894	823	.915	863	.936	903	.956
664	.822	704	.848	744	.872	784	.894	824	.916	864	.937	904	.956
665	0.823	705	0.848	745	0.872	785	0.895	825	0.916	865	0.937	905	0.957
666	.823	706	.849	746	.873	786	.895	826	.917	866	.938	906	.957
667	.824	707	.849	747	.873	787	.896	827	.918	867	.938	907	.958
668	.825	708	.850	748	.874	788	.897	828	.918	868	.939	908	.958
669	.825	709	.851	749	.874	789	.897	829	.919	869	.939	909	.959
670	0.826	710	0.851	750	0.875	790	0.898	830	0.919	870	0.940	910	0.959
671	.827	711	.852	751	.876	791	.898	831	.920	871	.940	911	.960
672	.827	712	.852	752	.876	792	.899	832	.920	872	.941	912	.960
673	.828	713	.853	753	.877	793	.899	833	.921	873	.941	913	.960
674	.829	714	.854	754	.877	794	.900	834	.921	874	.942	914	.961
675	0.829	715	0.854	755	0.878	795	0.900	835	0.922	875	0.942	915	0.961
676	.830	716	.855	756	.879	796	.901	836	.922	876	.943	916	.962
677	.831	717	.856	757	.879	797	.901	837	.923	877	.943	917	.962
678	.831	718	.856	758	.880	798	.902	838	.923	878	.943	918	.963
679	.832	719	.857	759	.880	799	.903	839	.924	879	.944	919	.963
680	0.833	720	0.857	760	0.881	800	0.903	840	0.924	880	0.944	920	0.964
681	.833	721	.858	761	.881	801	.904	841	.925	881	.945	921	.964
682	.834	722	.859	762	.882	802	.904	842	.925	882	.945	922	.965
683	.834	723	.859	763	.883	803	.905	843	.926	883	.946	923	.965
684	.835	724	.860	764	.883	804	.905	844	.926	884	.946	924	.966
685	0.836	725	0.860	765	0.884	805	0.906	845	0.927	885	0.947	925	0.966
686	.836	726	.861	766	.884	806	.906	846	.927	886	.947	926	.967
687	.837	727	.862	767	.885	807	.907	847	.928	887	.948	927	.967
688	.838	728	.862	768	.885	808	.907	848	.928	888	.948	928	.968
689	.838	729	.863	769	.886	809	.908	849	.929	889	.949	929	.968
690	0.839	730	0.863	770	0.886	810	0.908	850	0.929	890	0.949	930	0.968
691	.839	731	.864	771	.887	811	.909	851	.930	891	.950	931	.969
692	.840	732	.865	772	.888	812	.910	852	.930	892	.950	932	.969
693	.841	733	.865	773	.888	813	.910	853	.931	893	.951	933	.970
694	.841	734	.866	774	.889	814	.911	854	.931	894	.951	934	.970
695	0.842	735	0.866	775	0.889	815	0.911	855	0.932	895	0.952	935	0.971
696	.843	736	.867	776	.890	816	.912	856	.932	896	.952	936	.971
697	.843	737	.867	777	.890	817	.912	857	.933	897	.953	937	.972
698	.844	738	.868	778	.891	818	.913	858	.933	898	.953	938	.972
699	.844	739	.869	779	.892	819	.913	859	.934	899	.954	939	.973

TABLE I (*Continued*).
COMMON LOGARITHMS.

N	Log.	*N*	Log.	*N*	Log.	*N*	Log.	*N*	Log.	*N*	Log.
940	0.973	960	0.982	980	0.991	1000	0.000	1020	0.009	1040	0.017
941	.974	961	.983	981	.992	1001	.000	1021	.009	1041	.017
942	.974	962	.983	982	.992	1002	.001	1022	.009	1042	.018
943	.975	963	.984	983	.993	1003	.001	1023	.010	1043	.018
944	.975	964	.984	984	.993	1004	.002	1024	.010	1044	.019
945	0.975	965	0.985	985	0.993	1005	0.002	1025	0.011	1045	0.019
946	.976	966	.985	986	.994	1006	.003	1026	.011	1046	.020
947	.976	967	.985	987	.994	1007	.003	1027	.012	1047	.020
948	.977	968	.986	988	.995	1008	.003	1028	.012	1048	.020
949	.977	969	.986	989	.995	1009	.004	1029	.012	1049	.021
950	0.978	970	0.987	990	0.996	1010	0.004	1030	0.013	1050	0.021
951	.978	971	.987	991	.996	1011	.005	1031	.013	1051	.022
952	.979	972	.988	992	.997	1012	.005	1032	.014	1052	.022
953	.979	973	.988	993	.997	1013	.006	1033	.014	1053	.022
954	.980	974	.989	994	.997	1014	.006	1034	.015	1054	.023
955	0.980	975	0.989	995	0.998	1015	0.006	1035	0.015	1055	0.023
956	.980	976	.989	996	.998	1016	.007	1036	.015	1056	.024
957	.981	977	.990	997	.999	1017	.007	1037	.016	1057	.024
958	.981	978	.990	998	.999	1018	.008	1038	.016	1058	.024
959	.982	979	.991	999	.000	1019	.008	1039	.017	1059	.025

TABLE 2.

Trigonometric Logarithms.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
0.0	—	0.000	—	—	0.000	—	4.0	8.844	9.999	8.845	1.155	0.001	1.156
.1	7.242	.000	7.242	2.758	.000	2.758	.1	.854	.999	.855	.145	.001	.146
.2	.543	.000	.543	.457	.000	.457	.2	.865	.999	.866	.134	.001	.135
.3	.719	.000	.719	.281	.000	.281	.3	.875	.999	.876	.124	.001	.125
.4	.844	.000	.844	.156	.000	.156	.4	.885	.999	.886	.114	.001	.115
.5	7.941	0.000	7.941	2.059	0.000	2.059	.5	8.895	9.999	8.896	1.104	0.001	1.105
.6	8.020	.000	8.020	1.980	.000	1.980	.6	.904	.999	.906	.094	.001	.096
.7	.087	.000	.087	.913	.000	.913	.7	.913	.999	.915	.085	.001	.087
.8	.145	.000	.145	.855	.000	.855	.8	.923	.998	.924	.076	.002	.077
.9	.196	.000	.196	.804	.000	.804	.9	.932	.998	.933	.067	.002	.068
1.0	8.242	0.000	8.242	1.758	0.000	1.758	5.0	8.940	9.998	8.942	1.058	0.002	1.060
.1	.283	.000	.283	.717	.000	.717	.1	.949	.998	.951	.049	.002	.051
.2	.321	.000	.321	.679	.000	.679	.2	.957	.998	.959	.041	.002	.043
.3	.356	.000	.356	.644	.000	.644	.3	.966	.998	.967	.033	.002	.034
.4	.388	.000	.388	.612	.000	.612	.4	.974	.998	.976	.024	.002	.026
.5	8.418	0.000	8.418	1.582	0.000	1.582	.5	8.982	9.998	8.984	1.016	0.002	1.018
.6	.446	.000	.446	.554	.000	.554	.6	.989	.998	.991	.009	.002	.011
.7	.472	.000	.472	.528	.000	.528	.7	.997	.998	.999	.001	.002	.003
.8	.497	.000	.497	.503	.000	.503	.8	9.005	.998	9.007	0.993	.002	0.995
.9	.521	.000	.521	.479	.000	.479	.9	.012	.998	.014	.986	.002	.988
2.0	8.543	0.000	8.543	1.457	0.000	1.457	6.0	9.019	9.998	9.022	0.978	0.002	0.981
.1	.564	.000	.564	.436	.000	.436	.1	.026	.998	.029	.971	.002	.974
.2	.584	.000	.585	.415	.000	.416	.2	.033	.997	.036	.964	.003	.967
.3	.603	.000	.604	.396	.000	.397	.3	.040	.997	.043	.957	.003	.960
.4	.622	.000	.622	.378	.000	.378	.4	.047	.997	.050	.950	.003	.953
.5	8.640	0.000	8.640	1.360	0.000	1.360	.5	9.054	9.997	9.057	0.943	0.003	0.946
.6	.657	.000	.657	.343	.000	.343	.6	.060	.997	.063	.937	.003	.940
.7	.673	.000	.674	.326	.000	.327	.7	.067	.997	.070	.930	.003	.933
.8	.689	9.999	.689	.311	.001	.311	.8	.073	.997	.076	.924	.003	.927
.9	.704	.999	.705	.295	.001	.296	.9	.080	.997	.083	.917	.003	.920
3.0	8.719	9.999	8.719	1.281	0.001	1.281	7.0	9.086	9.997	9.089	0.911	0.003	0.914
.1	.733	.999	.734	.266	.001	.267	.1	.092	.997	.095	.905	.003	.908
.2	.747	.999	.747	.253	.001	.253	.2	.098	.997	.102	.898	.003	.902
.3	.760	.999	.761	.239	.001	.240	.3	.104	.996	.108	.892	.004	.896
.4	.773	.999	.774	.226	.001	.227	.4	.110	.996	.114	.886	.004	.890
.5	8.786	9.999	8.786	1.214	0.001	1.214	.5	9.116	9.996	9.119	0.881	0.004	0.884
.6	.798	.999	.799	.201	.001	.202	.6	.121	.996	.125	.875	.004	.879
.7	.810	.999	.811	.189	.001	.190	.7	.127	.996	.131	.869	.004	.873
.8	.821	.999	.822	.178	.001	.179	.8	.133	.996	.137	.863	.004	.867
.9	.833	.999	.834	.166	.001	.167	.9	.138	.996	.142	.858	.004	.862

TABLE 2 (*Continued*).

TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
8.0	9.144	9.996	9.148	0.852	0.004	0.856	12.0	9.318	9.990	9.327	0.673	0.010	0.682
.1	.149	.996	.153	.847	.004	.851	.1	.321	.990	.331	.669	.010	.679
.2	.154	.996	.159	.841	.004	.846	.2	.325	.990	.335	.665	.010	.675
.3	.159	.995	.164	.836	.005	.841	.3	.328	.990	.339	.661	.010	.672
.4	.165	.995	.169	.831	.005	.835	.4	.332	.990	.342	.658	.010	.668
.5	9.170	9.995	9.174	0.826	0.005	0.830	.5	9.335	9.990	9.346	0.654	0.010	0.665
.6	.175	.995	.180	.820	.005	.825	.6	.339	.989	.349	.651	.011	.661
.7	.180	.995	.185	.815	.005	.820	.7	.342	.989	.353	.647	.011	.658
.8	.185	.995	.190	.810	.005	.815	.8	.345	.989	.356	.644	.011	.655
.9	.190	.995	.195	.805	.005	.810	.9	.349	.989	.360	.640	.011	.651
9.0	9.194	9.995	9.200	0.800	0.005	0.806	13.0	9.352	9.989	9.363	0.637	0.011	0.648
.1	.199	.994	.205	.795	.006	.801	.1	.355	.989	.367	.633	.011	.645
.2	.204	.994	.209	.791	.006	.796	.2	.359	.988	.370	.630	.012	.641
.3	.208	.994	.214	.786	.006	.792	.3	.362	.988	.374	.626	.012	.638
.4	.213	.994	.219	.781	.006	.787	.4	.365	.988	.377	.623	.012	.635
.5	9.218	9.994	9.224	0.776	0.006	0.782	.5	9.368	9.988	9.380	0.620	0.012	0.632
.6	.222	.994	.228	.772	.006	.778	.6	.371	.988	.384	.616	.012	.629
.7	.227	.994	.233	.767	.006	.773	.7	.374	.987	.387	.613	.013	.626
.8	.231	.994	.237	.763	.006	.769	.8	.378	.987	.390	.610	.013	.622
.9	.235	.993	.242	.758	.007	.765	.9	.381	.987	.394	.606	.013	.619
10.0	9.240	9.993	9.246	0.754	0.007	0.760	14.0	9.384	9.987	9.397	0.603	0.013	0.616
.1	.244	.993	.251	.749	.007	.756	.1	.387	.987	.400	.600	.013	.613
.2	.248	.993	.255	.745	.007	.752	.2	.390	.987	.403	.597	.013	.610
.3	.252	.993	.259	.741	.007	.748	.3	.393	.986	.406	.594	.014	.607
.4	.257	.993	.264	.736	.007	.743	.4	.396	.986	.410	.590	.014	.604
.5	9.261	9.993	9.268	0.732	0.007	0.739	.5	9.399	9.986	9.413	0.587	0.014	0.601
.6	.265	.993	.272	.728	.007	.735	.6	.402	.986	.416	.584	.014	.598
.7	.269	.992	.276	.724	.008	.731	.7	.404	.986	.419	.581	.014	.596
.8	.273	.992	.280	.720	.008	.727	.8	.407	.985	.422	.578	.015	.593
.9	.277	.992	.285	.715	.008	.723	.9	.410	.985	.425	.575	.015	.590
11.0	9.281	9.992	9.289	0.711	0.008	0.719	15.0	9.413	9.985	9.428	0.572	0.015	0.587
.1	.284	.992	.293	.707	.008	.716	.1	.416	.985	.431	.569	.015	.584
.2	.288	.992	.297	.703	.008	.712	.2	.419	.985	.434	.566	.015	.581
.3	.292	.991	.301	.699	.009	.708	.3	.421	.984	.437	.563	.016	.579
.4	.296	.991	.305	.695	.009	.704	.4	.424	.984	.440	.560	.016	.576
.5	9.300	9.991	9.308	0.692	0.009	0.700	.5	9.427	9.984	9.443	0.557	0.016	0.573
.6	.303	.991	.312	.688	.009	.697	.6	.430	.984	.446	.554	.016	.570
.7	.307	.991	.316	.684	.009	.693	.7	.432	.983	.449	.551	.017	.568
.8	.311	.991	.320	.680	.009	.689	.8	.435	.983	.452	.548	.017	.565
.9	.314	.991	.324	.676	.009	.686	.9	.438	.983	.455	.545	.017	.562

TABLE 2 (*Continued*).
TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
16.0	9.440	9.983	9.457	0.543	0.017	0.560	20.0	9.534	9.973	9.561	0.439	0.027	0.466
.1	.443	.983	.460	.540	.017	.557	.1	.536	.973	.563	.437	.027	.464
.2	.446	.982	.463	.537	.018	.554	.2	.538	.972	.566	.434	.028	.462
.3	.448	.982	.466	.534	.018	.552	.3	.540	.972	.568	.432	.028	.460
.4	.451	.982	.469	.531	.018	.549	.4	.542	.972	.570	.430	.028	.458
.5	9.453	9.982	9.472	0.528	0.018	0.547	.5	9.544	9.972	9.573	0.427	0.028	0.456
.6	.456	.982	.474	.526	.018	.544	.6	.546	.971	.575	.425	.029	.454
.7	.458	.981	.477	.523	.019	.542	.7	.548	.971	.577	.423	.029	.452
.8	.461	.981	.480	.520	.019	.539	.8	.550	.971	.580	.420	.029	.450
.9	.463	.981	.483	.517	.019	.537	.9	.552	.970	.582	.418	.030	.448
17.0	9.466	9.981	9.485	0.515	0.019	0.534	21.0	9.554	9.970	9.584	0.416	0.030	0.446
.1	.468	.980	.488	.512	.020	.532	.1	.556	.970	.586	.414	.030	.444
.2	.471	.980	.491	.509	.020	.529	.2	.558	.970	.589	.411	.030	.442
.3	.473	.980	.493	.507	.020	.527	.3	.560	.969	.591	.409	.031	.440
.4	.476	.980	.496	.504	.020	.524	.4	.562	.969	.593	.407	.031	.438
.5	9.478	9.979	9.499	0.501	0.021	0.522	.5	9.564	9.969	9.595	0.405	0.031	0.436
.6	.481	.979	.501	.499	.021	.519	.6	.566	.968	.598	.402	.032	.434
.7	.483	.979	.504	.496	.021	.517	.7	.568	.968	.600	.400	.032	.432
.8	.485	.979	.507	.493	.021	.515	.8	.570	.968	.602	.398	.032	.430
.9	.488	.978	.509	.491	.022	.512	.9	.572	.967	.604	.396	.033	.428
18.0	9.490	9.978	9.512	0.488	0.022	0.510	22.0	9.574	9.967	9.606	0.394	0.033	0.426
.1	.492	.978	.514	.486	.022	.508	.1	.575	.967	.609	.391	.033	.425
.2	.495	.978	.517	.483	.022	.505	.2	.577	.967	.611	.389	.033	.423
.3	.497	.977	.519	.481	.023	.503	.3	.579	.966	.613	.387	.034	.421
.4	.499	.977	.522	.478	.023	.501	.4	.581	.966	.615	.385	.034	.419
.5	9.501	9.977	9.525	0.475	0.023	0.499	.5	9.583	9.966	9.617	0.383	0.034	0.417
.6	.504	.977	.527	.473	.023	.496	.6	.585	.965	.619	.381	.035	.415
.7	.506	.976	.530	.470	.024	.494	.7	.586	.965	.621	.379	.035	.414
.8	.508	.976	.532	.468	.024	.492	.8	.588	.965	.624	.376	.035	.412
.9	.510	.976	.535	.465	.024	.490	.9	.590	.964	.626	.374	.036	.410
19.0	9.513	9.976	9.537	0.463	0.024	0.487	23.0	9.592	9.964	9.628	0.372	0.036	0.408
.1	.515	.975	.539	.461	.025	.485	.1	.594	.964	.630	.370	.036	.406
.2	.517	.975	.542	.458	.025	.483	.2	.595	.963	.632	.368	.037	.405
.3	.519	.975	.544	.456	.025	.481	.3	.597	.963	.634	.366	.037	.403
.4	.521	.975	.547	.453	.025	.479	.4	.599	.963	.636	.364	.037	.401
.5	9.523	9.974	9.549	0.451	0.026	0.477	.5	9.601	9.962	9.638	0.362	0.038	0.399
.6	.526	.974	.552	.448	.026	.474	.6	.602	.962	.640	.360	.038	.398
.7	.528	.974	.554	.446	.026	.472	.7	.604	.962	.642	.358	.038	.396
.8	.530	.974	.556	.444	.026	.470	.8	.606	.961	.644	.356	.039	.394
.9	.532	.973	.559	.441	.027	.468	.9	.608	.961	.647	.353	.039	.392

TABLE 2 (*Continued*).
TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
24.0	9.609	9.961	9.649	0.351	0.039	0.391	28.0	9.672	9.946	9.726	0.274	0.054	0.328
.1	.611	.960	.651	.349	.040	.389	.1	.673	.946	.728	.272	.054	.327
.2	.613	.960	.653	.347	.040	.387	.2	.674	.945	.729	.271	.055	.326
.3	.614	.960	.655	.345	.040	.386	.3	.676	.945	.731	.269	.055	.324
.4	.616	.959	.657	.343	.041	.384	.4	.677	.944	.733	.267	.056	.323
.5	9.618	9.959	9.659	0.341	0.041	0.382	.5	9.679	9.944	9.735	0.265	0.056	0.321
.6	.619	.959	.661	.339	.041	.381	.6	.680	.943	.737	.263	.057	.320
.7	.621	.958	.663	.337	.042	.379	.7	.681	.943	.738	.262	.057	.319
.8	.623	.958	.665	.335	.042	.377	.8	.683	.943	.740	.260	.057	.317
.9	.624	.958	.667	.333	.042	.376	.9	.684	.942	.742	.258	.058	.316
25.0	9.626	9.957	9.669	0.331	0.043	0.374	29.0	9.686	9.942	9.744	0.256	0.058	0.314
.1	.628	.957	.671	.329	.043	.372	.1	.687	.941	.746	.254	.059	.313
.2	.629	.957	.673	.327	.043	.371	.2	.688	.941	.747	.253	.059	.312
.3	.631	.956	.675	.325	.044	.369	.3	.690	.941	.749	.251	.059	.310
.4	.632	.956	.677	.323	.044	.368	.4	.691	.940	.751	.249	.060	.309
.5	9.634	9.955	9.678	0.322	0.045	0.366	.5	9.692	9.940	9.753	0.247	0.060	0.308
.6	.636	.955	.680	.320	.045	.364	.6	.694	.939	.754	.246	.061	.306
.7	.637	.955	.682	.318	.045	.363	.7	.695	.939	.756	.244	.061	.305
.8	.639	.954	.684	.316	.046	.361	.8	.696	.938	.758	.242	.062	.304
.9	.640	.954	.686	.314	.046	.360	.9	.698	.938	.760	.240	.062	.302
26.0	9.642	9.954	9.688	0.312	0.046	0.358	30.0	9.699	9.938	9.761	0.239	0.062	0.301
.1	.643	.953	.690	.310	.047	.357	.1	.700	.937	.763	.237	.063	.300
.2	.645	.953	.692	.308	.047	.355	.2	.702	.937	.765	.235	.063	.298
.3	.646	.953	.694	.306	.047	.354	.3	.703	.936	.767	.233	.064	.297
.4	.648	.952	.696	.304	.048	.352	.4	.704	.936	.768	.232	.064	.296
.5	9.650	9.952	9.698	0.302	0.048	0.350	.5	9.705	9.935	9.770	0.230	0.065	.295
.6	.651	.951	.700	.300	.049	.349	.6	.707	.935	.772	.228	.065	.293
.7	.653	.951	.702	.298	.049	.347	.7	.708	.934	.774	.226	.066	.292
.8	.654	.951	.703	.297	.049	.346	.8	.709	.934	.775	.225	.066	.291
.9	.656	.950	.705	.295	.050	.344	.9	.711	.934	.777	.223	.066	.289
27.0	9.657	9.950	9.707	0.293	0.050	0.343	31.0	9.712	9.933	9.779	0.221	0.067	0.288
.1	.659	.949	.709	.291	.051	.341	.1	.713	.933	.780	.220	.067	.287
.2	.660	.949	.711	.289	.051	.340	.2	.714	.932	.782	.218	.068	.286
.3	.661	.949	.713	.287	.051	.339	.3	.716	.932	.784	.216	.068	.284
.4	.663	.948	.715	.285	.052	.337	.4	.717	.931	.786	.214	.069	.283
.5	9.664	9.948	9.716	0.284	0.052	0.336	.5	9.718	9.931	9.787	0.213	0.069	0.282
.6	.666	.948	.718	.282	.052	.334	.6	.719	.930	.789	.211	.070	.281
.7	.667	.947	.720	.280	.053	.333	.7	.721	.930	.791	.209	.070	.279
.8	.669	.947	.722	.278	.053	.331	.8	.722	.929	.792	.208	.071	.278
.9	.670	.946	.724	.276	.054	.330	.9	.723	.929	.794	.206	.071	.277

TABLE 2 (*Continued*).
TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
32.0	9.724	9.928	9.796	0.204	0.072	0.276	36.0	9.769	9.908	9.861	0.139	0.092	0.231
.1	.725	.928	.797	.203	.072	.275	.1	.770	.907	.863	.137	.093	.230
.2	.727	.927	.799	.201	.073	.273	.2	.771	.907	.864	.136	.093	.229
.3	.728	.927	.801	.199	.073	.272	.3	.772	.906	.866	.134	.094	.228
.4	.729	.927	.803	.197	.073	.271	.4	.773	.906	.868	.132	.094	.227
.5	9.730	9.926	9.804	0.196	0.074	0.270	.5	9.774	9.905	9.869	0.131	0.095	0.226
.6	.731	.926	.806	.194	.074	.269	.6	.775	.905	.871	.129	.095	.225
.7	.733	.925	.808	.192	.075	.267	.7	.776	.904	.872	.128	.096	.224
.8	.734	.925	.809	.191	.075	.266	.8	.777	.903	.874	.126	.097	.223
.9	.735	.924	.811	.189	.076	.265	.9	.778	.903	.876	.124	.097	.222
33.0	9.736	9.924	9.813	0.187	0.076	0.264	37.0	9.779	9.902	9.877	0.123	0.098	0.221
.1	.737	.923	.814	.186	.077	.263	.1	.780	.902	.879	.121	.098	.220
.2	.738	.923	.816	.184	.077	.262	.2	.781	.901	.880	.120	.099	.219
.3	.740	.922	.817	.183	.078	.260	.3	.782	.901	.882	.118	.099	.218
.4	.741	.922	.819	.181	.078	.259	.4	.783	.900	.883	.117	.100	.217
.5	9.742	9.921	9.821	0.179	0.079	0.258	.5	9.784	9.899	9.885	0.115	0.101	0.216
.6	.743	.921	.822	.178	.079	.257	.6	.785	.899	.887	.113	.101	.215
.7	.744	.920	.824	.176	.080	.256	.7	.786	.898	.888	.112	.102	.214
.8	.745	.920	.826	.174	.080	.255	.8	.787	.898	.890	.110	.102	.213
.9	.746	.919	.827	.173	.081	.254	.9	.788	.897	.891	.109	.103	.212
34.0	9.748	9.919	9.829	0.171	0.081	0.252	38.0	9.789	9.897	9.893	0.107	0.103	0.211
.1	.749	.918	.831	.169	.082	.251	.1	.790	.896	.894	.106	.104	.210
.2	.750	.918	.832	.168	.082	.250	.2	.791	.895	.896	.104	.105	.209
.3	.751	.917	.834	.166	.083	.249	.3	.792	.895	.897	.103	.105	.208
.4	.752	.917	.836	.164	.083	.248	.4	.793	.894	.899	.101	.106	.207
.5	9.753	9.916	9.837	0.163	0.084	0.247	.5	9.794	9.894	9.901	0.099	0.106	0.206
.6	.754	.915	.839	.161	.085	.246	.6	.795	.893	.902	.098	.107	.205
.7	.755	.915	.840	.160	.085	.245	.7	.796	.892	.904	.096	.108	.204
.8	.756	.914	.842	.158	.086	.244	.8	.797	.892	.905	.095	.108	.203
.9	.758	.914	.844	.156	.086	.242	.9	.798	.891	.907	.093	.109	.202
35.0	9.759	9.913	9.845	0.155	0.087	0.241	39.0	9.799	9.891	9.908	0.092	0.109	0.201
.1	.760	.913	.847	.153	.087	.240	.1	.800	.890	.910	.090	.110	.200
.2	.761	.912	.848	.152	.088	.239	.2	.801	.889	.911	.089	.111	.199
.3	.762	.912	.850	.150	.088	.238	.3	.802	.889	.913	.087	.111	.198
.4	.763	.911	.852	.148	.089	.237	.4	.803	.888	.915	.085	.112	.197
.5	9.764	9.911	9.853	0.147	0.089	0.236	.5	9.804	9.887	9.916	0.084	0.113	0.196
.6	.765	.910	.855	.145	.090	.235	.6	.804	.887	.918	.082	.113	.196
.7	.766	.910	.856	.144	.090	.234	.7	.805	.886	.919	.081	.114	.195
.8	.767	.909	.858	.142	.091	.233	.8	.806	.886	.921	.079	.114	.194
.9	.768	.909	.860	.140	.091	.232	.9	.807	.885	.922	.078	.115	.193

TABLE 2 (*Continued*).
TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
40.0	9.808	9.884	9.924	0.076	0.116	0.192	44.0	9.842	9.857	9.985	0.015	0.143	0.158
.1	.809	.884	.925	.075	.116	.191	.1	.843	.856	.986	.014	.144	.157
.2	.810	.883	.927	.073	.117	.190	.2	.843	.855	.988	.012	.145	.157
.3	.811	.882	.928	072	.118	.189	.3	.844	.855	.989	.011	.145	.156
.4	.812	.882	.930	.070	.118	.188	.4	.845	.854	.991	.009	.146	.155
.5	9.813	9.881	9.931	0.069	0.119	0.187	.5	9.846	9.853	9.992	0.008	0.147	0.154
.6	.813	.880	.933	.067	.120	.187	.6	.846	.852	.994	.006	.148	.154
.7	.814	.880	.935	.065	.120	.186	.7	.847	.852	.995	.005	.148	.153
.8	.815	.879	.936	.064	.121	.185	.8	.848	.851	.997	.003	.149	.152
.9	.816	.878	.938	.062	.122	.184	.9	.849	.850	.998	.002	.150	.151
41.0	9.817	9.878	9.939	0.061	0.122	0.183	45.0	9.849	9.849	0.000	0.000	0.150	0.151
.1	.818	.877	.941	.059	.123	.182	.1	.850	.849	.002	9.998	.151	.150
.2	.819	.876	.942	.058	.124	.181	.2	.851	.848	.003	.997	.152	.149
.3	.820	.876	.944	.056	.124	.180	.3	.852	.847	.005	.995	.153	.148
.4	.820	.875	.945	.055	.125	.180	.4	.852	.846	.006	.994	.154	.148
.5	9.821	9.874	9.947	0.053	0.126	0.179	.5	9.853	9.846	0.008	9.992	0.154	0.147
.6	.822	.874	.948	.052	.126	.178	.6	.854	.845	.009	.991	.155	.146
.7	.823	.873	.950	.050	.127	.177	.7	.855	.844	.011	.989	.156	.145
.8	.824	.872	.951	.049	.128	.176	.8	.855	.843	.012	.988	.157	.145
.9	.825	.872	.953	.047	.128	.175	.9	.856	.843	.014	.986	.157	.144
42.0	9.826	9.871	9.954	0.046	0.129	0.174	46.0	9.857	9.842	0.015	9.985	0.158	0.143
.1	.826	.870	.956	.044	.130	.174	.1	.858	.841	.017	.983	.159	.142
.2	.827	.870	.957	.043	.130	.173	.2	.858	.840	.018	.982	.160	.142
.3	.828	.869	.959	.041	.131	.172	.3	.859	.839	.020	.980	.161	.141
.4	.829	.868	.961	.039	.132	.171	.4	.860	.839	.021	.979	.161	.140
.5	9.830	9.868	9.962	0.038	0.132	0.170	.5	9.861	9.838	0.023	9.977	0.162	0.139
.6	.831	.867	.964	.036	.133	.169	.6	.861	.837	.024	.976	.163	.139
.7	.831	.866	.965	.035	.134	.169	.7	.862	.836	.026	.974	.164	.138
.8	.832	.866	.967	.033	.134	.168	.8	.863	.835	.027	.973	.165	.137
.9	.833	.865	.968	.032	.135	.167	.9	.863	.835	.029	.971	.165	.137
43.0	9.834	9.864	9.970	0.030	0.136	0.166	47.0	9.864	9.834	0.030	9.970	0.166	0.136
.1	.835	.863	.971	.029	.137	.165	.1	.865	.833	.032	.968	.167	.135
.2	.835	.863	.973	.027	.137	.165	.2	.866	.832	.033	.967	.168	.134
.3	.836	.862	.974	.026	.138	.164	.3	.866	.831	.035	.965	.169	.134
.4	.837	.861	.976	.024	.139	.163	.4	.867	.831	.036	.964	.169	.133
.5	9.838	9.861	9.977	0.023	0.139	0.162	.5	9.868	9.830	0.038	9.962	0.170	0.132
.6	.839	.860	.979	.021	.140	.161	.6	.868	.829	.039	.961	.171	.132
.7	.839	.859	.980	.020	.141	.161	.7	.869	.828	.041	.959	.172	.131
.8	.840	.858	.982	.018	.142	.160	.8	.870	.827	.043	.957	.173	.130
.9	.841	.858	.983	.017	.142	.159	.9	.870	.826	.044	.956	.174	.130

TABLE 2 (*Continued*).
TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
48.0	9.871	9.826	0.046	9.954	0.174	0.129	52.0	9.897	9.789	0.107	9.893	0.211	0.103
.1	.872	.825	.047	.953	.175	.128	.1	.897	.788	.109	.891	.212	.103
.2	.872	.824	.049	.951	.176	.128	.2	.898	.787	.110	.890	.213	.102
.3	.873	.823	.050	.950	.177	.127	.3	.898	.786	.112	.888	.214	.102
.4	.874	.822	.052	.948	.178	.126	.4	.899	.785	.113	.887	.215	.101
.5	9.874	9.821	0.053	9.947	0.179	0.126	.5	9.899	9.784	0.115	9.885	0.216	0.101
.6	.875	.820	.055	.945	.180	.125	.6	.900	.783	.117	.883	.217	.100
.7	.876	.820	.056	.944	.180	.124	.7	.901	.782	.118	.882	.218	.099
.8	.876	.819	.058	.942	.181	.124	.8	.901	.781	.120	.880	.219	.099
.9	.877	.818	.059	.941	.182	.123	.9	.902	.780	.121	.879	.220	.098
49.0	9.878	9.817	0.061	9.939	0.183	0.122	53.0	9.902	9.779	0.123	9.877	0.221	0.098
.1	.878	.816	.062	.938	.184	.122	.1	.903	.778	.124	.876	.222	.097
.2	.879	.815	.064	.936	.185	.121	.2	.903	.777	.126	.874	.223	.097
.3	.880	.814	.065	.935	.186	.120	.3	.904	.776	.128	.872	.224	.096
.4	.880	.813	.067	.933	.187	.120	.4	.905	.775	.129	.871	.225	.095
.5	9.881	9.813	0.069	9.931	0.187	0.119	.5	9.905	9.774	0.131	9.869	0.226	0.095
.6	.882	.812	.070	.930	.188	.118	.6	.906	.773	.132	.868	.227	.094
.7	.882	.811	.072	.928	.189	.118	.7	.906	.772	.134	.866	.228	.094
.8	.883	.810	.073	.927	.190	.117	.8	.907	.771	.136	.864	.229	.093
.9	.884	.809	.075	.925	.191	.116	.9	.907	.770	.137	.863	.230	.093
50.0	9.884	9.808	0.076	9.924	0.192	0.116	54.0	9.908	9.769	0.139	9.861	0.231	0.092
.1	.885	.807	.078	.922	.193	.115	.1	.909	.768	.140	.860	.232	.091
.2	.886	.806	.079	.921	.194	.114	.2	.909	.767	.142	.858	.233	.091
.3	.886	.805	.081	.919	.195	.114	.3	.910	.766	.144	.856	.234	.090
.4	.887	.804	.082	.918	.196	.113	.4	.910	.765	.145	.855	.235	.090
.5	9.887	9.804	0.084	9.916	0.196	0.113	.5	9.911	9.764	0.147	9.853	0.236	0.089
.6	.888	.803	.085	.915	.197	.112	.6	.911	.763	.148	.852	.237	.089
.7	.889	.802	.087	.913	.198	.111	.7	.912	.762	.150	.850	.238	.088
.8	.889	.801	.089	.911	.199	.111	.8	.912	.761	.152	.848	.239	.088
.9	.890	.800	.090	.910	.200	.110	.9	.913	.760	.153	.847	.240	.087
51.0	9.891	9.799	0.092	9.908	0.201	0.109	55.0	9.913	9.759	0.155	9.845	0.241	0.087
.1	.891	.798	.093	.907	.202	.109	.1	.914	.758	.156	.844	.242	.086
.2	.892	.797	.095	.905	.203	.108	.2	.914	.756	.158	.842	.244	.086
.3	.892	.796	.096	.904	.204	.108	.3	.915	.755	.160	.840	.245	.085
.4	.893	.795	.098	.902	.205	.107	.4	.915	.754	.161	.839	.246	.085
.5	9.894	9.794	0.099	9.901	0.206	0.106	.5	9.916	9.753	0.163	9.837	0.247	0.084
.6	.894	.793	.101	.899	.207	.106	.6	.917	.752	.164	.836	.248	.083
.7	.895	.792	.103	.897	.208	.105	.7	.917	.751	.166	.834	.249	.083
.8	.895	.791	.104	.896	.209	.105	.8	.918	.750	.168	.832	.250	.082
.9	.896	.790	.106	.894	.210	.104	.9	.918	.749	.169	.831	.251	.082

TABLE 2 (*Continued*).
TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
56.0	9.919	9.748	0.171	9.829	0.252	0.081	60.0	9.938	9.699	0.239	9.761	0.301	0.062
.1	.919	.746	.173	.827	.254	.081	.1	.938	.698	.240	.760	.302	.062
.2	.920	.745	.174	.826	.255	.080	.2	.938	.696	.242	.758	.304	.062
.3	.920	.744	.176	.824	.256	.080	.3	.939	.695	.244	.756	.305	.061
.4	.921	.743	.178	.822	.257	.079	.4	.939	.694	.246	.754	.306	.061
.5	9.921	9.742	0.179	9.821	0.258	0.079	.5	9.940	9.692	0.247	9.753	0.308	0.060
.6	.922	.741	.181	.819	.259	.078	.6	.940	.691	.249	.751	.309	.060
.7	.922	.740	.183	.817	.260	.078	.7	.941	.690	.251	.749	.310	.059
.8	.923	.738	.184	.816	.262	.077	.8	.941	.688	.253	.747	.312	.059
.9	.923	.737	.186	.814	.263	.077	.9	.941	.687	.254	.746	.313	.059
57.0	9.924	9.736	0.187	9.813	0.264	0.076	61.0	9.942	9.686	0.256	9.744	0.314	0.058
.1	.924	.735	.189	.811	.265	.076	.1	.942	.684	.258	.742	.316	.058
.2	.925	.734	.191	.809	.266	.075	.2	.943	.683	.260	.740	.317	.057
.3	.925	.733	.192	.808	.267	.075	.3	.943	.681	.262	.738	.319	.057
.4	.926	.731	.194	.806	.269	.074	.4	.943	.680	.263	.737	.320	.057
.5	9.926	9.730	0.196	9.804	0.270	0.074	.5	9.944	9.679	0.265	9.735	0.321	0.056
.6	.927	.729	.197	.803	.271	.073	.6	.944	.677	.267	.733	.323	.056
.7	.927	.728	.199	.801	.272	.073	.7	.945	.676	.269	.731	.324	.055
.8	.927	.727	.201	.799	.273	.073	.8	.945	.674	.271	.729	.326	.055
.9	.928	.725	.203	.797	.275	.072	.9	.946	.673	.272	.728	.327	.054
58.0	9.928	9.724	0.204	9.796	0.276	0.072	62.0	9.946	9.672	0.274	9.726	0.328	0.054
.1	.929	.723	.206	.794	.277	.071	.1	.946	.670	.276	.724	.330	.054
.2	.929	.722	.208	.792	.278	.071	.2	.947	.669	.278	.722	.331	.053
.3	.930	.721	.209	.791	.279	.070	.3	.947	.667	.280	.720	.333	.053
.4	.930	.719	.211	.789	.281	.070	.4	.948	.666	.282	.718	.334	.052
.5	9.931	9.718	0.213	9.787	0.282	0.069	.5	9.948	9.664	0.284	9.716	0.336	0.052
.6	.931	.717	.214	.786	.283	.069	.6	.948	.663	.285	.715	.337	.052
.7	.932	.716	.216	.784	.284	.068	.7	.949	.661	.287	.713	.339	.051
.8	.932	.714	.218	.782	.286	.068	.8	.949	.660	.289	.711	.340	.051
.9	.933	.713	.220	.780	.287	.067	.9	.949	.659	.291	.709	.341	.051
59.0	9.933	9.712	0.221	9.779	0.288	0.067	63.0	9.950	9.657	0.293	9.707	0.343	0.050
.1	.934	.711	.223	.777	.289	.066	.1	.950	.656	.295	.705	.344	.050
.2	.934	.709	.225	.775	.291	.066	.2	.951	.654	.297	.703	.346	.049
.3	.934	.708	.226	.774	.292	.066	.3	.951	.653	.298	.702	.347	.049
.4	.935	.707	.228	.772	.293	.065	.4	.951	.651	.300	.700	.349	.049
.5	9.935	9.705	0.230	9.770	0.295	0.065	.5	9.952	9.650	0.302	9.698	0.350	0.048
.6	.936	.704	.232	.768	.296	.064	.6	.952	.648	.304	.696	.352	.048
.7	.936	.703	.233	.767	.297	.064	.7	.953	.646	.306	.694	.354	.047
.8	.937	.702	.235	.765	.298	.063	.8	.953	.645	.308	.692	.355	.047
.9	.937	.700	.237	.763	.300	.063	.9	.953	.643	.310	.690	.357	.047

TABLE 2 (*Continued*).
TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
64.0	9.954	9.642	0.312	9.688	0.358	0.046	68.0	9.967	9.574	0.394	9.606	0.426	0.033
.1	.954	.640	.314	.686	.360	.046	.1	.967	.572	.396	.604	.428	.033
.2	.954	.639	.316	.684	.361	.046	.2	.968	.570	.398	.602	.430	.032
.3	.955	.637	.318	.682	.363	.045	.3	.968	.568	.400	.600	.432	.032
.4	.955	.636	.320	.680	.364	.045	.4	.968	.566	.402	.598	.434	.032
.5	9.955	9.634	0.322	9.678	0.366	0.045	.5	9.969	9.564	0.405	9.595	0.436	0.031
.6	.956	.632	.323	.677	.368	.044	.6	.969	.562	.407	.593	.438	.031
.7	.956	.631	.325	.675	.369	.044	.7	.969	.560	.409	.591	.440	.031
.8	.957	.629	.327	.673	.371	.043	.8	.970	.558	.411	.589	.442	.030
.9	.957	.628	.329	.671	.372	.043	.9	.970	.556	.414	.586	.444	.030
65.0	9.957	9.626	0.331	9.669	0.374	0.043	69.0	9.970	9.554	0.416	9.584	0.446	0.030
.1	.958	.624	.333	.667	.376	.042	.1	.970	.552	.418	.582	.448	.030
.2	.958	.623	.335	.665	.377	.042	.2	.971	.550	.420	.580	.450	.029
.3	.958	.621	.337	.663	.379	.042	.3	.971	.548	.423	.577	.452	.029
.4	.959	.619	.339	.661	.381	.041	.4	.971	.546	.425	.575	.454	.029
.5	9.959	9.618	0.341	9.659	0.382	0.041	.5	9.972	9.544	0.427	9.573	0.456	0.028
.6	.959	.616	.343	.657	.384	.041	.6	.972	.542	.430	.570	.458	.028
.7	.960	.614	.345	.655	.386	.040	.7	.972	.540	.432	.568	.460	.028
.8	.960	.613	.347	.653	.387	.040	.8	.972	.538	.434	.566	.462	.028
.9	.960	.611	.349	.651	.389	.040	.9	.973	.536	.437	.563	.464	.027
66.0	9.961	9.609	0.351	9.649	0.391	0.039	70.0	9.973	9.534	0.439	9.561	0.466	0.027
.1	.961	.608	.353	.647	.392	.039	.1	.973	.532	.441	.559	.468	.027
.2	.961	.606	.356	.644	.394	.039	.2	.974	.530	.444	.556	.470	.026
.3	.962	.604	.358	.642	.396	.038	.3	.974	.528	.446	.554	.472	.026
.4	.962	.602	.360	.640	.398	.038	.4	.974	.526	.448	.552	.474	.026
.5	9.962	9.601	0.362	9.638	0.399	0.038	.5	9.974	9.523	0.451	9.549	0.477	0.026
.6	.963	.599	.364	.636	.401	.037	.6	.975	.521	.453	.547	.479	.025
.7	.963	.597	.366	.634	.403	.037	.7	.975	.519	.456	.544	.481	.025
.8	.963	.595	.368	.632	.405	.037	.8	.975	.517	.458	.542	.483	.025
.9	.964	.594	.370	.630	.406	.036	.9	.975	.515	.461	.539	.485	.025
67.0	9.964	9.592	0.372	9.628	0.408	0.036	71.0	9.976	9.513	0.463	9.537	0.487	0.024
.1	.964	.590	.374	.626	.410	.036	.1	.976	.510	.465	.535	.490	.024
.2	.965	.588	.376	.624	.412	.035	.2	.976	.508	.468	.532	.492	.024
.3	.965	.586	.379	.621	.414	.035	.3	.976	.506	.470	.530	.494	.024
.4	.965	.585	.381	.619	.415	.035	.4	.977	.504	.473	.527	.496	.023
.5	9.966	9.583	0.383	9.617	0.417	0.034	.5	9.977	9.501	0.475	9.525	0.499	0.023
.6	.966	.581	.385	.615	.419	.034	.6	.977	.499	.478	.522	.501	.023
.7	.966	.579	.387	.613	.421	.034	.7	.977	.497	.481	.519	.503	.023
.8	.967	.577	.389	.611	.423	.033	.8	.978	.495	.483	.517	.505	.022
.9	.967	.575	.391	.609	.425	.033	.9	.978	.492	.486	.514	.508	.022

TABLE 2 (*Continued*).
TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
72.0	9.978	9.490	0.488	9.512	0.510	0.022	76.0	9.987	9.384	0.603	9.397	0.616	0.013
.1	.978	.488	.491	.509	.512	.022	.1	.987	.381	.606	.394	.619	.013
.2	.979	.485	.493	.507	.515	.021	.2	.987	.378	.610	.390	.622	.013
.3	.979	.483	.496	.504	.517	.021	.3	.987	.374	.613	.387	.626	.013
.4	.979	.481	.499	.501	.519	.021	.4	.988	.371	.616	.384	.629	.012
.5	9.979	9.478	0.501	9.499	0.522	0.021	.5	9.988	9.368	0.620	9.380	0.632	0.012
.6	.980	.476	.504	.496	.524	.020	.6	.988	.365	.623	.377	.635	.012
.7	.980	.473	.507	.493	.527	.020	.7	.988	.362	.626	.374	.638	.012
.8	.980	.471	.509	.491	.529	.020	.8	.988	.359	.630	.370	.641	.012
.9	.980	.468	.512	.488	.532	.020	.9	.989	.355	.633	.367	.645	.011
73.0	9.981	9.466	0.515	9.485	0.534	0.019	77.0	9.989	9.352	0.637	9.363	0.648	0.011
.1	.981	.463	.517	.483	.537	.019	.1	.989	.349	.640	.360	.651	.011
.2	.981	.461	.520	.480	.539	.019	.2	.989	.345	.644	.356	.655	.011
.3	.981	.458	.523	.477	.542	.019	.3	.989	.342	.647	.353	.658	.011
.4	.982	.456	.526	.474	.544	.018	.4	.989	.339	.651	.349	.661	.011
.5	9.982	9.453	0.528	9.472	0.547	0.018	.5	9.990	9.335	0.654	9.346	0.665	0.010
.6	.982	.451	.531	.469	.549	.018	.6	.990	.332	.658	.342	.668	.010
.7	.982	.448	.534	.466	.552	.018	.7	.990	.328	.661	.339	.672	.010
.8	.982	.446	.537	.463	.554	.018	.8	.990	.325	.665	.335	.675	.010
.9	.983	.443	.540	.460	.557	.017	.9	.990	.321	.669	.331	.679	.010
74.0	9.983	9.440	0.543	9.457	0.560	0.017	78.0	9.990	9.318	0.673	9.327	0.682	0.010
.1	.983	.438	.545	.455	.562	.017	.1	.991	.314	.676	.324	.686	.009
.2	.983	.435	.548	.452	.565	.017	.2	.991	.311	.680	.320	.689	.009
.3	.983	.432	.551	.449	.568	.017	.3	.991	.307	.684	.316	.693	.009
.4	.984	.430	.554	.446	.570	.016	.4	.991	.303	.688	.312	.697	.009
.5	9.984	9.427	0.557	9.443	0.573	0.016	.5	9.991	9.300	0.692	9.308	0.700	0.009
.6	.984	.424	.560	.440	.576	.016	.6	.991	.296	.695	.305	.704	.009
.7	.984	.421	.563	.437	.579	.016	.7	.991	.292	.699	.301	.708	.009
.8	.985	.419	.566	.434	.581	.015	.8	.992	.288	.703	.297	.712	.008
.9	.985	.416	.569	.431	.584	.015	.9	.992	.284	.707	.293	.716	.008
75.0	9.985	9.413	0.572	9.428	0.587	0.015	79.0	9.992	9.281	0.711	9.289	0.719	0.008
.1	.985	.410	.575	.425	.590	.015	.1	.992	.277	.715	.285	.723	.008
.2	.985	.407	.578	.422	.593	.015	.2	.992	.273	.720	.280	.727	.008
.3	.986	.404	.581	.419	.596	.014	.3	.992	.269	.724	.276	.731	.008
.4	.986	.402	.584	.416	.598	.014	.4	.993	.265	.728	.272	.735	.007
.5	9.986	9.399	0.587	9.413	0.601	0.014	.5	9.993	9.261	0.732	9.268	0.739	0.007
.6	.986	.396	.590	.410	.604	.014	.6	.993	.257	.736	.264	.743	.007
.7	.986	.393	.594	.406	.607	.014	.7	.993	.252	.741	.259	.748	.007
.8	.987	.390	.597	.403	.610	.013	.8	.993	.248	.745	.255	.752	.007
.9	.987	.387	.600	.400	.613	.013	.9	.993	.244	.749	.251	.756	.007

TABLE 2 (*Continued*).
TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
80.0	9.993	9.240	0.754	9.246	0.760	0.007	84.0	9.998	9.019	0.978	9.022	0.981	0.002
.1	.993	.235	.758	.242	.765	.007	.1	.998	.012	.986	.014	.988	.002
.2	.994	.231	.763	.237	.769	.006	.2	.998	.005	.993	.007	.995	.002
.3	.994	.227	.767	.233	.773	.006	.3	.998	8.997	1.001	8.999	1.003	.002
.4	.994	.222	.772	.228	.778	.006	.4	.998	.989	.009	.991	.011	.002
.5	9.994	9.218	0.776	9.224	0.782	0.006	.5	9.998	8.982	1.016	8.984	1.018	0.002
.6	.994	.213	.781	.219	.787	.006	.6	.998	.974	.024	.976	.026	.002
.7	.994	.208	.786	.214	.792	.006	.7	.998	.966	.033	.967	.034	.002
.8	.994	.204	.791	.209	.796	.006	.8	.998	.957	.041	.959	.043	.002
.9	.994	.199	.795	.205	.801	.006	.9	.998	.949	.049	.951	.051	.002
81.0	9.995	9.194	0.800	9.200	0.806	0.005	85.0	9.998	8.940	1.058	8.942	1.060	0.002
.1	.995	.190	.805	.195	.810	.005	.1	.998	.932	.067	.933	.068	.002
.2	.995	.185	.810	.190	.815	.005	.2	.998	.923	.076	.924	.077	.002
.3	.995	.180	.815	.185	.820	.005	.3	.999	.913	.085	.915	.087	.001
.4	.995	.175	.820	.180	.825	.005	.4	.999	.904	.094	.906	.096	.001
.5	9.995	9.170	0.826	9.174	0.830	0.005	.5	9.999	8.895	1.104	8.896	1.105	0.001
.6	.995	.165	.831	.169	.835	.005	.6	.999	.885	.114	.886	.115	.001
.7	.995	.159	.836	.164	.841	.005	.7	.999	.875	.124	.876	.125	.001
.8	.996	.154	.841	.159	.846	.004	.8	.999	.865	.134	.866	.135	.001
.9	.996	.149	.847	.153	.851	.004	.9	.999	.854	.145	.855	.146	.001
82.0	9.996	9.144	0.852	9.148	0.856	0.004	86.0	9.999	8.844	1.155	8.845	1.156	0.001
.1	.996	.138	.858	.142	.862	.004	.1	.999	.833	.166	.834	.167	.001
.2	.996	.133	.863	.137	.867	.004	.2	.999	.821	.178	.822	.179	.001
.3	.996	.127	.869	.131	.873	.004	.3	.999	.810	.189	.811	.190	.001
.4	.996	.121	.875	.125	.879	.004	.4	.999	.798	.201	.799	.202	.001
.5	9.996	9.116	0.881	9.119	0.884	0.004	.5	9.999	8.786	1.214	8.786	1.214	0.001
.6	.996	.110	.886	.114	.890	.004	.6	.999	.773	.226	.774	.227	.001
.7	.996	.104	.892	.108	.896	.004	.7	.999	.760	.239	.761	.240	.001
.8	.997	.098	.898	.102	.902	.003	.8	.999	.747	.253	.747	.253	.001
.9	.997	.092	.905	.095	.908	.003	.9	.999	.733	.266	.734	.267	.001
83.0	9.997	9.086	0.911	9.089	0.914	0.003	87.0	9.999	8.719	1.281	8.719	1.281	0.001
.1	.997	.080	.917	.083	.920	.003	.1	.999	.704	.295	.705	.296	.001
.2	.997	.073	.924	.076	.927	.003	.2	.999	.689	.311	.689	.311	.001
.3	.997	.067	.930	.070	.933	.003	.3	0.000	.673	.326	.674	.327	.000
.4	.997	.060	.937	.063	.940	.003	.4	.000	.657	.343	.657	.343	.000
.5	9.997	9.054	0.943	9.057	0.946	0.003	.5	0.000	8.640	1.360	8.640	1.360	0.000
.6	.997	.047	.950	.050	.953	.003	.6	.000	.622	.378	.622	.378	.000
.7	.997	.040	.957	.043	.960	.003	.7	.000	.603	.396	.604	.397	.000
.8	.997	.033	.964	.036	.967	.003	.8	.000	.584	.415	.585	.416	.000
.9	.998	.026	.971	.029	.974	.002	.9	.000	.564	.436	.564	.436	.000

TABLE 2 (*Continued*).

TRIGONOMETRIC LOGARITHMS.

°	sin	cos	tan	cot	sec	cosec	°	sin	cos	tan	cot	sec	cosec
88.0	0.000	8.543	1.457	8.543	1.457	0.000	89.0	0.000	8.242	1.758	8.242	1.758	0.000
.1	.000	.521	.479	.521	.479	.000	.1	.000	.196	.804	.196	.804	.000
.2	.000	.497	.503	.497	.503	.000	.2	.000	.145	.855	.145	.855	.000
.3	.000	.472	.528	.472	.528	.000	.3	.000	.087	.913	.087	.913	.000
.4	.000	.446	.554	.446	.554	.000	.4	.000	.020	.980	.020	.980	.000
.5	0.000	8.418	1.582	8.418	1.582	0.000	.5	0.000	7.941	2.059	7.941	2.059	0.000
.6	.000	.388	.612	.388	.612	.000	.6	.000	.844	2.156	.844	.156	.000
.7	.000	.356	.644	.356	.644	.000	.7	.000	.719	.281	.719	.281	.000
.8	.000	.321	.679	.321	.679	.000	.8	.000	.543	.457	.543	.457	.000
.9	.000	.283	.717	.283	.717	.000	.9	.000	.242	.758	.242	.758	.000

TABLE 3.

Addition Logarithms.

$\log a - \log b = D.$ $\log a + A = \log (a + b).$

D	A	D	A	D	A	D	A
0.000–.001	0.301	0.084–5	0.261	0.177–9	0.221	0.286–7	0.181
.002–3	.300	.086–7	.260	.180–1	.220	.288–90	.180
.004–5	.299	.088–9	.259	.182–4	.219	.291–3	.179
.006–7	.298	.090–1	.258	.185–7	.218	.294–6	.178
.008–9	.297	.092–4	.257	.188–9	.217	.297–9	.177
0.010–1	0.296	0.095–6	0.256	0.190–2	0.216	0.300–2	0.176
.012–3	.295	.097–8	.255	.193–4	.215	.303–5	.175
.014–5	.294	.099–.100	.254	.195–7	.214	.306–8	.174
.016–7	.293	.101–3	.253	.198–9	.213	.309–11	.173
.018–9	.292	.104–5	.252	.200–2	.212	.312–4	.172
0.020–1	0.291	0.106–7	0.251	0.203–5	0.211	0.315–8	0.171
.022–3	.290	.108–10	.250	.206–7	.210	.319–21	.170
.024–5	.289	.111–.112	.249	.208–10	.209	.322–4	.169
.026–7	.288	.113–4	.248	.211–12	.208	.325–7	.168
.028–9	.287	.115–6	.247	.213–15	.207	.328–30	.167
0.030–1	0.286	0.117–9	0.246	0.216–18	0.206	0.331–3	0.166
.032–3	.285	.120–1	.245	.219–20	.205	.334–6	.165
.034–5	.284	.122–3	.244	.221–3	.204	.337–9	.164
.036–7	.283	.124–6	.243	.224–6	.203	.340–3	.163
.038–9	.282	.127–8	.242	.227–8	.202	.344–6	.162
0.040–2	0.281	0.129–30	0.241	0.229–31	0.201	0.347–9	0.161
.043–4	.280	.131–3	.240	.232–4	.200	.350–2	.160
.045–6	.279	.134–5	.239	.235–6	.199	.353–6	.159
.047–8	.278	.136–7	.238	.237–9	.198	.357–9	.158
.049–50	.277	.138–40	.237	.240–2	.197	.360–2	.157
0.051–2	0.276	0.141–2	0.236	0.243–5	0.196	0.363–5	0.156
.053–4	.275	.143–5	.235	.246–7	.195	.366–9	.155
.055–6	.274	.146–7	.234	.248–50	.194	.370–2	.154
.057–9	.273	.148–9	.233	.251–3	.193	.373–6	.153
.060–1	.272	.150–2	.232	.254–6	.192	.377–9	.152
0.062–3	0.271	0.153–4	0.231	0.257–9	0.191	0.380–2	0.151
.064–5	.270	.155–7	.230	.260–1	.190	.383–6	.150
.066–7	.269	.158–9	.229	.262–4	.189	.387–9	.149
.068–9	.268	.160–2	.228	.265–7	.188	.390–3	.148
.070–2	.267	.163–4	.227	.268–70	.187	.394–6	.147
0.073–4	0.266	0.165–7	0.226	0.271–3	0.186	0.397–.400	0.146
.075–6	.265	.168–9	.225	.274–6	.185	.401–3	.145
.077–8	.264	.170–1	.224	.277–9	.184	.404–7	.144
.079–80	.263	.172–4	.223	.280–2	.183	.408–10	.143
.081–3	.262	.175–6	.222	.283–5	.182	.411–4	.142

TABLE 3 (*Continued*).

ADDITION LOGARITHMS.

$\log a - \log b = D.$ $\log a + A = \log (a + b).$

D	A	D	A	D	A	D	A
0.415–7	0.141	0.557–60	0.106	0.748–53	0.071	1.058–69	0.036
.418–21	.140	.561–5	.105	.754–60	.070	.070–82	.035
.422–5	.139	.566–70	.104	.761–7	.069	.083–95	.034
.426–8	.138	.571–4	.103	.768–74	.068	.096–.109	.033
.429–32	.137	.575–9	.102	.775–81	.067	.110–23	.032
0.433–6	0.136	0.580–4	0.101	0.782–8	0.066	1.124–38	0.031
.437–40	.135	.585–9	.100	.789–95	.065	.139–53	.030
.441–3	.134	.590–4	.099	.796–.802	.064	.154–68	.029
.444–7	.133	.595–9	.098	.803–.810	.063	.169–84	.028
.448–51	.132	.600–4	.097	.811–7	.062	.185–.201	.027
0.452–5	0.131	0.605–9	0.096	0.818–25	0.061	1.202–18	0.026
.456–9	.130	.610–4	.095	.826–33	.060	.219–36	.025
.460–3	.129	.615–9	.094	.834–41	.059	.237–54	.024
.464–6	.128	.620–4	.093	.842–9	.058	.255–74	.023
.467–70	.127	.625–9	.092	.850–7	.057	.275–94	.022
0.471–4	0.126	0.630–5	0.091	0.858–65	0.056	1.295–.315	0.021
.475–8	.125	.636–40	.090	.866–73	.055	.316–37	.020
.479–82	.124	.641–5	.089	.874–82	.054	.338–61	.019
.483–6	.123	.646–51	.088	.883–91	.053	.362–85	.018
.487–91	.122	.652–6	.087	.892–9	.052	.386–.412	.017
0.492–5	0.121	0.657–62	0.086	0.900–8	0.051	1.413–39	0.016
.496–9	.120	.663–7	.085	.909–18	.050	.440–69	.015
.500–3	.119	.668–73	.084	.919–27	.049	.470–.500	.014
.504–7	.118	.674–9	.083	.928–37	.048	.501–34	.013
.508–11	.117	.680–5	.082	.938–46	.047	.535–71	.012
0.512–6	0.116	0.686–91	0.081	0.947–56	0.046	1.572–.611	0.011
.517–20	.115	.692–7	.080	.957–66	.045	.612–55	.010
.521–4	.114	.698–.703	.079	.967–77	.044	.656–.704	.009
.525–9	.113	.704–9	.078	.978–87	.043	.705–58	.008
.530–3	.112	.710–5	.077	.988–98	.042	.759–.821	.007
0.534–8	0.111	0.716–21	0.076	0.999–1.009	0.041	1.822–94	0.006
.539–42	.110	.722–7	.075	1.010–21	.040	.895–.982	.005
.543–6	.109	.728–34	.074	.022–32	.039	1.983–2.091	.004
.547–51	.108	.735–40	.073	.033–44	.038	2.092–.238	.003
.552–6	.107	.741–7	.072	.045–57	.037	.239–.460	.002
						2.461–.938	0.001
						2.939–	0.000

TABLE 4.

Subtraction Logarithms.

$\log a - \log b = D.$ $\log a - S = \log (a - b).$

D	S	D	S	D	S	D	S
0.000	∞	0.040	1.056	0.080	0.774	0.120	0.617
.001	2.638	.041	.045	.081	.769	.121	.614
.002	.338	.042	.035	.082	.764	.122	.611
.003	.162	.043	.026	.083	.760	.123	.608
.004	2.038	.044	.016	.084	.755	.124	.605
0.005	1.941	0.045	1.007	0.085	0.750	0.125	0.602
.006	.863	.046	0.998	.086	.746	.126	.599
.007	.796	.047	.989	.087	.741	.127	.596
.008	.739	.048	.980	.088	.737	.128	.593
.009	.688	.049	.972	.089	.732	.129	.590
0.010	1.643	0.050	0.964	0.090	0.728	0.130	0.587
.011	.602	.051	.955	.091	.723	.131	.584
.012	.565	.052	.948	.092	.719	.132	.582
.013	.530	.053	.940	.093	.715	.133	.579
.014	.499	.054	.932	.094	.711	.134	.576
0.015	1.469	0.055	0.925	0.095	0.707	0.135	0.573
.016	.442	.056	.917	.096	.703	.136	.570
.017	.416	.057	.910	.097	.699	.137	.568
.018	.391	.058	.903	.098	.695	.138	.565
.019	.368	.059	.896	.099	.691	.139	.562
0.020	1.347	0.060	0.889	0.100	0.687	0.140	0.560
.021	.326	.061	.883	.101	.683	.141	.557
.022	.306	.062	.876	.102	.679	.142	.555
.023	.288	.063	.870	.103	.675	.143	.552
.024	.270	.064	.863	.104	.672	.144	.549
0.025	1.252	0.065	0.857	.105	0.668	0.145	0.547
.026	.236	.066	.851	.106	.664	.146	.544
.027	.220	.067	.845	.107	.661	.147	.542
.028	.205	.068	.839	.108	.657	.148	.539
.029	.190	.069	.833	.109	.654	.149	.537
0.030	1.176	0.070	0.827	0.110	0.650	0.150	0.535
.031	.162	.071	.822	.111	.647	.151	.532
.032	.149	.072	.816	.112	.643	.152	.530
.033	.136	.073	.810	.113	.640	.153	.527
.034	.123	.074	.805	.114	.637	.154	.525
0.035	1.111	0.075	0.800	0.115	0.633	0.155	0.523
.036	.099	.076	.794	.116	.630	.156	.520
.037	.088	.077	.789	.117	.627	.157	.518
.038	.077	.078	.784	.118	.624	.158	.516
.039	.066	.079	.779	.119	.620	.159	.513

TABLE 4 (*Continued*).

SUBTRACTION LOGARITHMS.

$\log a - \log b = D.$ $\log a - S = \log (a - b).$

D	S	D	S	D	S	D	S
0.160	0.511	0.200	0.433	0.240	0.372	0.280	0.323
.161	.509	.201	.431	.241	.371	.281	.322
.162	.507	.202	.430	.242	.369	.282	.321
.163	.505	.203	.428	.243	.368	.283	.320
.164	.502	.204	.426	.244	.367	.284	.319
0.165	0.500	0.205	0.425	0.245	0.365	0.285	0.318
.166	.498	.206	.423	.246	.364	.286	.317
.167	.496	.207	.421	.247	.363	.287	.316
.168	.494	.208	.420	.248	.361	.288	.314
.169	.492	.209	.418	.249	.360	.289	.313
0.170	0.490	0.210	0.416	0.250	0.359	0.290	0.312
.171	.487	.211	.415	.251	.358	.291	.311
.172	.485	.212	.413	.252	.356	.292	.310
.173	.483	.213	.412	.253	.355	.293	.309
.174	.481	.214	.410	.254	.354	.294	.308
0.175	0.479	0.215	0.408	0.255	0.353	0.295	.307
.176	.477	.216	.407	.256	.351	.296	.306
.177	.475	.217	.405	.257	.350	.297	.305
.178	.473	.218	.404	.258	.349	.298	.304
.179	.471	.219	.402	.259	.348	.299	.303
0.180	0.469	0.220	0.401	0.260	0.346	0.300	0.302
.181	.467	.221	.399	.261	.345	.301	.301
.182	.466	.222	.398	.262	.344	.302	.300
.183	.464	.223	.396	.263	.343	.303	.299
.184	.462	.224	.395	.264	.342	.304	.298
0.185	0.460	0.225	0.393	0.265	0.340	0.305	0.297
.186	.458	.226	.392	.266	.339	.306	.296
.187	.456	.227	.390	.267	.338	.307	.295
.188	.454	.228	.389	.268	.337	.308	.294
.189	.452	.229	.387	.269	.336	.309	.293
0.190	0.451	0.230	0.386	0.270	0.334	0.310	0.292
.191	.449	.231	.385	.271	.333	.311	.291
.192	.447	.232	.383	.272	.332	.312	.290
.193	.445	.233	.382	.273	.331	.313	.289
.194	.443	.234	.380	.274	.330	.314	.288
0.195	0.442	0.235	0.379	0.275	0.329	0.315–6	0.287
.196	.440	.236	.378	.276	.328	.317	.286
.197	.438	.237	.376	.277	.326	.318	.285
.198	.436	.238	.375	.278	.325	.319	.284
.199	.435	.239	.373	.279	.324	.320	.283

TABLE 4 (*Continued*).

SUBTRACTION LOGARITHMS.

$\log a - \log b = D.$ $\log a - S = \log (a - b).$

D	S	D	S	D	S	D	S
0.321	0.282	0.369–70	0.242	0.429–30	0.202	0.506–7	0.162
.322	.281	.371	.241	.431–2	.201	.508–10	.161
.323	.280	.372	.240	.433	.200	.511–2	.160
.324	.279	.373–4	.239	.434–5	.199	.513–4	.159
.325	.278	.375	.238	.436–7	.198	.515–6	.158
0.326–7	0.277	0.376	0.237	0.438	0.197	0.517–9	0.157
.328	.276	.377–8	.236	.439–40	.196	.520–1	.156
.329	.275	.379	.235	.441–2	.195	.522–3	.155
.330	.274	.380–1	.234	.443–4	.194	.524–6	.154
.331	.273	.382	.233	.445–6	.193	.527–8	.153
0.332	0.272	0.383	0.232	0.447	0.192	0.529–30	0.152
.333	.271	.384–5	.231	.448–9	.191	.531–3	.151
.334–5	.270	.386	.230	.450–51	.190	.534–5	.150
.336	.269	.387–8	.229	.452–3	.189	.536–8	.149
.337	.268	.389	.228	.454–5	.188	.539–40	.148
0.338	0.267	0.390–1	0.227	0.456–7	0.187	0.541–3	0.147
.339	.266	.392	.226	.458	.186	.544–5	.146
.340	.265	.393	.225	.459–60	.185	.546–8	.145
.341–2	.264	.394–5	.224	.461–2	.184	.549–50	.144
.343	.263	.396	.223	.463–4	.183	.551–3	.143
0.344	0.262	0.397–8	0.222	0.465–6	0.182	0.554–5	0.142
.345	.261	.399	.221	.467–8	.181	.556–8	.141
.346	.260	.400–1	.220	.469–70	.180	.559–61	.140
.347–8	.259	.402–3	.219	.471–2	.179	.562–3	.139
.349	.258	.404	.218	.473–4	.178	.564–6	.138
0.350	0.257	0.405–6	0.217	0.475–6	0.177	0.567–9	0.137
.351	.256	.407	.216	.477–8	.176	.570–1	.136
.352–3	.255	.408–9	.215	.479–80	.175	.572–4	.135
.354	.254	.410	.214	.481–2	.174	.575–7	.134
.355	.253	.411–2	.213	.483–4	.173	.578–80	.133
0.356	0.252	0.413	0.212	0.485–6	0.172	0.581–2	0.132
.357–8	.251	.414–5	.211	.487–8	.171	.583–5	.131
.359	.250	.416–7	.210	.489–90	.170	.586–8	.130
.360	.249	.418	.209	.491–2	.169	.589–91	.129
.361–2	.248	.419–20	.208	.493–4	.168	.592–4	.128
0.363	0.247	0.421–2	0.207	0.495–6	0.167	0.595–7	0.127
.364	.246	.423	.206	.497–9	.166	.598–.600	.126
.365–6	.245	.424–5	.205	.500–1	.165	.601–3	.125
.367	.244	.426–7	.204	.502–3	.164	.604–6	.124
.368	.243	.428	.203	.504–5	.163	.607–9	.123

TABLE 4 (*Continued*).

SUBTRACTION LOGARITHMS.

$\log a - \log b = D.$ $\log a - S = \log (a - b).$

D	S	D	S	D	S	D	S
0.610–12	0.122	0.718–21	0.092	0.873–9	0.062	1.143–55	0.032
.613–5	.121	.722–5	.091	.880–5	.061	.156–68	.031
.616–8	.120	.726–9	.090	.886–92	.060	.169–82	.030
.619–21	.119	.730–4	.089	.893–9	.059	.183–97	.029
.622–5	.118	.735–8	.088	.900–6	.058	.198–.212	.028
0.626–8	0.117	0.739–43	0.087	0.907–13	0.057	1.213–27	0.027
.629–31	.116	.744–7	.086	.914–20	.056	.228–43	.026
.632–4	.115	.748–52	.085	.921–8	.055	.244–60	.025
.635–8	.114	.753–7	.084	.929–35	.054	.261–78	.024
.639–41	.113	.758–61	.083	.936–43	.053	.279–96	.023
0.642–5	0.112	0.762–6	0.082	0.944–51	0.052	1.297–.316	0.022
.646–8	.111	.767–71	.081	.952–9	.051	.317–36	.021
.649–51	.110	.772–6	.080	.960–7	.050	.337–57	.020
.652–5	.109	.777–81	.079	.968–76	.049	.358–79	.019
.656–9	.108	.782–6	.078	.977–84	.048	.380–.403	.018
0.660–2	0.107	0.787–91	0.077	0.985–93	0.047	1.404–28	0.017
.663–6	.106	.792–7	.076	.994–1.002	.046	.429–55	.016
.667–9	.105	.798–.802	.075	1.003–11	.045	.456–83	.015
.670–3	.104	.803–7	.074	.012–20	.044	.484–.514	.014
.674–7	.103	.808–13	.073	.021–30	.043	.515–47	.013
0.678–81	0.102	0.814–8	0.072	1.031–40	0.042	1.548–82	0.012
.682–4	.101	.819–24	.071	.041–50	.041	.583–.621	.011
.685–8	.100	.825–30	.070	.051–60	.040	.622–.664	.010
.689–92	.099	.831–5	.069	.061–71	.039	.665–.712	.009
.693–6	.098	.836–41	.068	.072–82	.038	.713–.766	.008
0.697–.700	0.097	0.842–7	0.067	1.083–93	0.037	1.767–.828	0.007
.701–4	.096	.848–53	.066	.094–.105	.036	1.829–.900	.006
.705–8	.095	.854–60	.065	.106–17	.035	.901–.986	.005
.709–12	.094	.861–6	.064	.118–29	.034	1.987–2.095	.004
.713–7	.093	.867–72	.063	.130–42	.033	2.096–.241	.003
						2.242–.462	.002
						2.463–.938	0.001

TABLE 5.

Giving M and N.

For North Latitudes (Read Down).

C.	log M	log N		C.	log M	log N		C.	log M	log N	
0°	—	—	180°								
1	9.699	3.360	179	41°	9.991	1.957	139°	81°	9.999	1.783	99°
2	.824	3.138	178	42	.992	.948	138	82	.999	.782	98
3	.875	2.992	177	43	.992	.940	137	83	.999	.781	97
4	.903	.884	176	44	.992	.932	139	84	.999	.780	96
5	.921	.797	175	45	.993	.925	135	85	9.999	.779	95
6	9.933	2.724	174	46	9.993	1.918	134	86	0.000	1.779	94
7	.942	.662	173	47	.993	.911	133	87	.000	.779	93
8	.949	.608	172	48	.993	.904	132	88	.000	.778	92
9	.955	.561	171	49	.994	.897	131	89	.000	.778	91
10	.959	.518	170	50	.994	.891	130	90	.000	.778	90
11	9.963	2.478	169	51	9.994	1.885	129	91	0.000	1.778	89
12	.966	.443	168	52	.994	.879	128	92	.000	.779	88
13	.968	.410	167	53	.994	.873	127	93	.000	.779	87
14	.971	.380	166	54	.995	.867	126	94	.001	.779	86
15	.973	.351	165	55	.995	.862	125	95	.001	.780	85
16	9.974	2.325	164	56	9.995	1.857	124	96	0.001	1.781	84
17	.976	.300	163	57	.995	.852	123	97	.001	.782	83
18	.977	.277	162	58	.995	.847	122	98	.001	.783	82
19	.979	.255	161	59	.996	.843	121	99	.001	.784	81
20	.980	.234	160	60	9.996	1.838	120	100	.001	.785	80
21	9.981	2.214	159	61	9.996	1.834	119	101	0.002	1.787	79
22	.982	.195	158	62	.996	.830	118	102	.002	.789	78
23	.983	.177	157	63	.996	.826	117	103	.002	.790	77
24	.983	.160	156	64	.996	.823	116	104	.002	.792	76
25	.984	.144	155	65	.997	.819	115	105	.002	.794	75
26	9.985	2.129	154	66	9.997	1.816	114	106	0.002	1.796	74
27	.985	.114	153	67	.997	.813	113	107	.002	.799	73
28	.986	.099	152	68	.997	.809	112	108	.003	.801	72
29	.987	.086	151	69	.997	.807	111	109	.003	.804	71
30	.987	.073	150	70	.997	.804	110	110	.003	.807	70
31	9.988	2.060	149	71	9.997	1.801	109	111	0.003	1.809	69
32	.988	.048	148	72	.998	.799	108	112	.003	.813	68
33	.989	.036	147	73	.998	.796	107	113	.003	.816	67
34	.989	.025	146	74	.998	.794	106	114	.003	.819	66
35	.989	.014	145	75	.998	.792	105	115	.004	.823	65
36	9.990	2.004	144	76	9.998	1.790	104	116	0.004	1.826	64
37	.990	1.994	143	77	.998	.789	103	117	.004	.830	63
38	.990	.984	142	78	.998	.787	102	118	.004	.834	62
39	.991	.975	141	79	.999	.785	101	119	.004	.838	61
40	.991	.966	140	80	.999	.784	100	120	0.004	1.843	60
	log M	log N	C.		log M	log N	C.		log M	log N	C.

For South Latitudes (Read Up).

TABLE 5 (*Continued*).

Giving M and N.

For North Latitudes (Read Down).

C.	log M	log N		C.	log M	log N		C.	log M	log N	
121°	0.005	1.847	59°	161°	0.023	2.277	19°	201°	0.020	2.234	339°
122	.005	.852	58	162	.024	.300	18	202	.019	.214	338
123	.005	.857	57	163	.026	.325	17	203	.018	.195	337
124	.005	.862	56	164	.027	.351	16	204	.017	.177	336
125	.005	.867	55	165	.029	.380	15	205	.017	.160	335
126	0.006	1.873	54	166	0.032	2.410	14	206	0.016	2.144	334
127	.006	.879	53	167	.034	.443	13	207	.015	.129	333
128	.006	.885	52	168	.037	.478	12	208	.015	.114	332
129	.006	.891	51	169	.041	.518	11	209	.014	.099	331
130	.006	.897	50	170	.045	.561	10	210	.013	.086	330
131	0.007	1.904	49	171	0.051	2.608	9	211	0.013	2.073	329
132	.007	.911	48	172	.058	.662	8	212	.012	.060	328
133	.007	.918	47	173	.067	.724	7	213	.012	.048	327
134	.007	.925	46	174	.079	.797	6	214	.011	.036	326
135	.008	.932	45	175	.097	.884	5	215	.011	.025	325
136	0.008	1.940	44	176	0.125	2.992	4	216	0.011	2.014	324
137	.008	.948	43	177	.176	3.138	3	217	.010	2.004	323
138	.009	.957	42	178	0.301	3.360	2	218	.010	1.994	322
139	.009	.966	41	179	—	—	1	219	.010	.984	321
140	.009	.975	40	180	—	—	0	220	.009	.975	320
141	0.010	1.984	39	181	—	—	359	221	0.009	1.966	319
142	.010	1.994	38	182	0.301	3.360	358	222	.009	.957	318
143	.010	2.004	37	183	.176	3.138	357	223	.008	.948	317
144	.011	.014	36	184	.125	2.992	356	224	.008	.940	316
145	.011	.025	35	185	.097	.884	355	225	.008	.932	315
146	0.011	2.036	34	186	0.079	2.797	354	226	0.007	1.925	314
147	.012	.048	33	187	.067	.724	353	227	.007	.918	313
148	.012	.060	32	188	.058	.662	352	228	.007	.911	312
149	.013	.073	31	189	.051	.608	351	229	.007	.904	311
150	.013	.086	30	190	.045	.561	350	230	.006	.897	310
151	0.014	2.099	29	191	0.041	2.518	349	231	0.006	1.891	309
152	.015	.114	28	192	.037	.478	348	232	.006	.885	308
153	.015	.129	27	193	.034	.443	347	233	.006	.879	307
154	.016	.144	26	194	.032	.410	346	234	.006	.873	306
155	.017	.160	25	195	.029	.380	345	235	.005	.867	305
156	0.017	2.177	24	196	0.027	2.351	344	236	0.005	1.862	304
157	.018	.195	23	197	.026	.325	343	237	.005	.857	303
158	.019	.214	22	198	.024	.230	342	238	.005	.852	302
159	.020	.234	21	199	.023	.277	341	239	.005	.847	301
160	.021	.255	20	200	.021	.255	340	240	0.004	1.843	300
	log M	log N	C.		log M	log N	C.		log M	log N	C.

For South Latitudes (Read Up).

TABLE 5 (*Continued*).
GIVING M AND N.
For North Latitudes (Read Down).

C.	log M	log N		C.	log M	log N		C.	log M	log N	
241°	0.004	1.838	299°	281°	9.999	1.785	259°	321°	9.991	1.975	219°
242	.004	.834	298	282	.998	.787	258	322	.990	.984	218
243	.004	.830	297	283	.998	.789	257	323	.990	1.994	217
244	.004	.826	296	284	.998	.790	256	324	.990	2.004	216
245	.004	.823	295	285	.998	.792	255	325	.989	.014	215
246	0.003	1.819	294	286	9.998	1.794	254	326	9.989	2.025	214
247	.003	.816	293	287	.998	.796	253	327	.989	.036	213
248	.003	.813	292	288	.998	.799	252	328	.988	.048	212
249	.003	.809	291	289	.997	.801	251	329	.988	.060	211
250	.003	.807	290	290	.997	.804	250	330	.987	.073	210
251	0.003	1.804	289	291	9.997	1.807	249	331	9.987	2.086	209
252	.003	.801	288	292	.997	.809	248	332	.986	.099	208
253	.002	.799	287	293	.997	.813	247	333	.985	.114	207
254	.002	.796	286	294	.997	.816	246	334	.985	.129	206
255	.002	.794	285	295	.997	.819	245	335	.984	.144	205
256	0.002	1.792	284	296	9.996	1.823	244	336	9.983	2.160	204
257	.002	.790	283	297	.996	.826	243	337	.983	.177	203
258	.002	.789	282	298	.996	.830	242	338	.982	.195	202
259	.002	.787	281	299	.996	.834	241	339	.981	.214	201
260	.001	.785	280	300	9.996	1.838	240	340	.980	.234	200
261	0.001	1.784	279	301	9.996	1.843	239	341	9.979	2.255	199
262	.001	.783	278	302	.995	.847	238	342	.977	.277	198
263	.001	.782	277	303	.995	.852	237	343	.976	.300	197
264	.001	.781	276	304	.995	.857	236	344	.974	.325	196
265	.001	.780	275	305	.995	.862	235	345	.973	.351	195
266	0.001	1.779	274	306	9.995	1.867	234	346	9.971	2.380	194
267	.000	.779	273	307	.994	.873	233	347	.968	.410	193
268	.000	.779	272	308	.994	.879	232	348	.966	.443	192
269	.000	.778	271	309	.994	.885	231	349	.963	.478	191
270	.000	.778	270	310	.994	.891	230	350	.959	.518	190
271	0.000	1.778	269	311	9.994	1.897	229	351	9.955	2.561	189
272	.000	.778	268	312	.993	.904	228	352	.949	.608	188
273	.000	.779	267	313	.993	.911	227	353	.942	.662	187
274	0.000	.779	266	314	.993	.918	226	354	.933	.724	186
275	9.999	.779	265	315	.993	.925	225	355	.921	.797	185
276	9.999	1.780	264	316	9.992	1.932	224	356	9.903	2.884	184
277	.999	.781	263	317	.992	.940	223	357	.875	2.992	183
278	.999	.782	262	318	.992	.948	222	358	.824	3.138	182
279	.999	.783	261	319	.991	.957	221	359	.699	3.360	181
280	.999	.784	260	320	.991	.966	220	360	—	—	180
	log M	log N	C.		log M	log N	C.		log M	log N	C.

For South Latitudes (Read Up).

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